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TECHNICAL REPORT 68253

THE APPLICATION OF VISUAL LOBE SEARCH THEORY TO AIR TO GROUND TARGET DETECTION

by

L. J. Smith, B.Sc.

with an Appendix by A. R. Runnalls

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FARNBOROUGH HANTS

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SUMMARY

Details are given of how a mathematical model, based on search theory and target size and contrast, can be programmed for a digital computer to enable the cumulative probability of detection of a ground target from the air to be determined as a function of range. This is applied to cover both visual and televisual viewing for flat targets and for targets of fixed presented area. An account of some of the difficulties involved is included, together with justification of some of the simplifications found possible.

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1 INTRODUCTION

The task of searching for and detecting from the air a target situated somewhere on the ground is a complex one to represent analytically, but nevertheless it forms an important part of research into target acquisition. A theoretical model of searching based on visual lobes* and glimpsing has been reviewed in Ref.1. This model considers the case of an observer approaching a finite search area within which a target of given size and contrast is situated and enables the probability of detecting the target in a single glimpse at any range to be obtained. These single glimpse probabilities may then be combined to give the cumulative probability of having detected the target by that range from the start of the run. In this paper, the theoretical model of search has been applied to the case of air to ground target detection and details are given showing how the theoretical concepts have been adapted and simplified to enable numerical results to be obtained. Two main simplifications to the theory are established - (i) the area of a visual lobe may be calculated by assuming that it is an ellipse, and (ii) the average visual lobe for the whole search area can be taken to be the lobe associated with the glimpse to the centre of the search area. Various complications and difficulties have been encountered in the application of the theory, especially regarding the calculation of the extreme ends of the lobe, and steps taken to overcome these are discussed. Due to the nature of the equations involved they can only be solved iteratively and most of the calculations were carried out on a digital computer. (The Mercury computer at R.A.E., Farnborough.)

2 SEARCH THEORY

The theory reviewed in Ref.1 leads to a simple expression for the cumulative probability of seeing a target after searching for it with 'N' glimpses directed randomly at the search areas, viz:

$$P_N = 1 - \prod_{n=1}^{n=N} (1 - p_n) \qquad (1)$$

where p_n is the probability of seeing the target with the nth glimpse.

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* See Ref.1 for a full discussion of these concepts. In the present study a visual lobe is taken to be the effective area or volume within which a target would always be seen and is calculated from the '50% frequency of seeing' contour. Natural search is assumed to be composed of a number of discrete glimpses, each corresponding to a given visual lobe size.

This equation presumes that if a target is caught in a glimpse it is seen immediately. This, in turn, implies that the probability of seeing the target if the observer is looking directly at it i.e. the foveal* probability of seeing the target, is either zero or unity and that there is no gradual increase in this probability from zero to unity as the size of the visual stimulus increases. In Ref.1, an extension to equation (1) is suggested to account for cases where the target is not seen every time even with foveal vision or extended search, viz.

$$P_N = p_f \left[1 - \prod_{n=1}^{n=N} (1 - p_n) \right] \qquad (2)$$

where p_f is the probability of seeing the target when the observer is looking at it, and p_n is slightly modified from that in equation (1). This modification will be incorporated into future search programmes, but for the present, with one exception, equation (1) above has proved adequate for most assessment problems. The exception to the use of equation (1) has been when a television camera is approaching a search area and the field of view of the camera is steadily cutting off an increasing part of the search area. This problem is analysed in Appendix A as an extension to equation (1). The most convenient computational expression is derived from equation (A-9) and is

$$P_N - P_{N-1} = p_n \frac{A_n}{A} \left[\prod_{1}^{N-1} [1 - p_n] \right] \qquad (3)$$

where A is the total search area and A_n is the search area within the field of view at any given glimpse.

In equation (1), the value of p_n used is basically a derivation from

$$p_n = \frac{a}{A}, \qquad (4)$$

where a is the area of the visual lobe associated with the nth glimpse, and A is the search area. The derivation allows for the fact that when a is large compared with A, there is a high probability that much of a is wasted outside the search area. It often takes the form of

$$p = \frac{a}{(A + a)} \qquad (5)$$

---

* The fovea is the part of the retina which allows for greatest resolution at normal daylight light levels. It is the part used when looking directly at an object.

but in Ref.1 a more correct form is derived, viz:

$$p = \left(\sqrt{\frac{a}{A}} - \frac{a}{4A}\right)^2, \tag{6a}$$

with the expression

$$p = \left(\frac{x}{X} - \frac{x^2}{4X^2}\right)\left(\frac{y}{Y} - \frac{y^2}{4Y^2}\right) \tag{6b}$$

being used when the lobes and search areas are not symmetrical but have dimensions x, y and X, Y respectively: in this case, the lobes must be assumed to be rectangular in a rectangular search area. Each half of the expression (6b) has a limit of validity when $x = 2X$ or $y = 2Y$, and for $x > 2X$ or $y > 2Y$ the corresponding half of equation (6) should be made equal to unity.

The derivation of the exact expression for glimpse probability for half of equation (6) i.e. for search along a line is given in Appendix B. The differences between the two methods of calculating single glimpse probability (equations 5 and 6a) are shown in Fig.1 as a function of the ratio lobe area/search area (a/A).

In the succeeding sub-sections a broad outline is given of the calculation of glimpse probabilities as an introduction to the details of the succeeding sections.

2.1 Visual and televisual search

Consider an observer in an aircraft that is flying towards a fixed, rectangular search area on the ground, somewhere within which a target of given size and contrast is situated. The observer may be using direct vision or looking at a television screen. The aircraft may either be flying at a fixed altitude or may be diving onto the centre of the search area at a constant speed and dive angle. For fixed altitude flight the perspective of the target may change during the overflight and this should be allowed for in the calculation of target size at any particular range; the two extreme cases are (1) a flat target (i.e. the plane of the target is in the plane of the search area) and (2) the case where the plane of the target is normal to the sight-line. The second case may also be taken to apply to the dive approach for which the target perspective does not change.

Before starting any calculations, a range must be estimated beyond which the probability of seeing the target is zero; the aircraft may then be assumed to fly in from this range. This range is the foveal detection range i.e. the distance between the observer and target such that the target placed anywhere in the search area could just not be detected if the observer were looking straight at it. This maximum detection range is assumed to be a sharp dividing line between visible and not visible according to the assumption implicit in equation (1) and is calculated by equating the apparent target contrast available at the observer's eye to the contrast required to detect the target; this calculation is described in Appendix C. The contrast required by the eye is obtained from an empirical threshold curve derived from some rather idealized laboratory experiments as described in Ref.1.

Theory assumes that an observer searches for a target in a number of discrete glimpses each occupying a finite time called the glimpse time. During this time the observer moves closer to the search area by a distance equal to the aircraft velocity times the glimpse time.

When the observer glimpses to any part of the search area there is associated with his glimpse a volume of revolution about the axis of his glimpse called a visual lobe within which the target can be seen. Since this paper is concerned with air to ground vision, the intersection of this volume with the ground plane defines the area within which a target can be seen in a single glimpse and this area will subsequently be referred to as a visual lobe. The boundary of this visual lobe is determined by the maximum angle off the sight line, θ at which the given target could still be seen, the value of which is governed by the amount of excess contrast available at the observer's eye over that required for foveal detection at the same range. The value of θ is also a function of target size, as seen by the observer, and is obtained from J. H. Taylor's data[2] on the off-axis performance of the eye which are depicted in Fig.2. In the programmes, linear approximations to the data in Fig.2 have been used as described in Appendix D.2.

With direct vision a visual lobe is normally calculated in terms of ground coordinates; with vision through television, however, one has the choice of ground coordinates or screen coordinates and in terms of visual lobe sizes the two possibilities are wholly equivalent.

It is pointed out in Ref.1 and also in section 6 ahead that in some instances it is very likely that lobe sizes are smaller than those predicted

from the laboratory data of J. H. Taylor, particularly when the target is in a complex background. To allow for this it is possible to divide these Taylor calculated values of θ by a constant, but no real information is at present available on the magnitude of this factor.

In order to calculate the average glimpse probability for targets which might appear anywhere in the search area, theory requires an average lobe size for the whole search area. In practice, the greatest variation in lobe areas occurs in the direction of flight and for most purposes an average of lobe sizes along the central axis of the search area in the direction of flight is sufficient. In section 5.2 ahead, it is also shown that for many purposes the lobe size for a glimpse directed centrally in the search area is a sufficiently good approximation.

With a calculated lobe area, or average lobe area for the whole search area, the glimpse probability may be calculated according to equation (7), and from this, the cumulative probability of having seen the target from the start of the run is calculated according to equation (1).

The process of calculating the glimpse probability is repeated at successively shorter ranges until either some predetermined value is reached or until the observer is immediately above the centre of the search area. From these calculations a cumulative detection probability distribution curve can be plotted against range.

The only complications introduced by television into the above sequence of calculations are the effects of blurring or lack of resolution, and its restricted field of view.

The succeeding sections follow the steps required in the calculation of visual search performance from the estimation of starting ranges for the calculation, through the calculation of lobe dimensions to the estimation of lobe area and glimpse probabilities. The visual situation is considered first in each section and then the additional factors introduced by television are presented. The approximations used to simplify the problem, and the difficulties encountered are referred to in the relevant sections.

3 CALCULATION OF FOVEAL DETECTION RANGE AT THE START OF A RUN

The starting range for the start of the search calculations is required to be that range beyond which there is no chance of seeing the target. This is the foveal detection range, assumed to be, to the approximation of

equation (1), a sharp dividing line between seeing and not seeing the target with foveal vision. This detection range is determined by target contrast and size and the visual and televisual cases are treated separately below.

3.1 Visual

The contrast available is the apparent contrast and is given by the atmospheric attenuation of the contrast equation. Throughout this paper the Blackwell 8 position search in six seconds threshold curve has been used to describe the eye's foveal performance but with an allowance for the addition of a constant (the degradation constant) to $\log_{10}$ contrast values to convert the laboratory threshold curve into one applicable to a practical viewing situation. The equations and method of solution are given in Appendix C.1, while the basic, unfactored threshold curve is shown in Fig.3.

3.2 Televisual

In the televisual case also, the foveal detection range is obtained by equating and solving iteratively for range the expressions giving the available contrast and that required for detection. These are given in detail in Appendix C.2 but are basically described as follows. The inherent contrast of the target is attenuated by the atmosphere and reaches the TV camera as an apparent contrast. In passing through the camera-monitor system this apparent contrast is attenuated by the camera and enhanced by the monitor to give an image contrast on the screen. The television system blurs the detail of the picture, but this is only partly accounted for in the theory by the attenuation of the contrast of small detail according to the frequency response curve of the camera, see, for example, Fig.4. In practice, the eye would be badly affected by looking at blurred detail, and in the theory the difficulty of specifying how badly the eye would be affected in looking for particular targets in particular backgrounds is circumvented by assuming that the observer always views at the viewing distance at which the screen image appears 'just sharp'. Comparison between television systems may then be made by scaling the viewing distance according to the resolution of the television to maintain this 'just sharp' condition. This aspect of visual theory, of predicting detection performance as a combined function of display and eye resolution needs more experimental investigation.

4 CALCULATION OF THE SHAPE AND SIZE OF A VISUAL LOBE

Two methods of calculation are used depending whether direct vision or television is being considered.

4.1 Direct vision

As a first step in the calculation of the area of the visual lobe in the plane of the search area, equations are derived in Appendix D.1 that define the shape of the lobe associated with a glimpse to any part of the search area. The geometry of the situation is depicted in Fig.5. In this figure, φ is the depression angle of the glimpse, θ is the angle of vision off the foveal axis, i.e. the maximum angle off at which the target can still be seen, and R_g and β are the polar coordinates of the circumference of the visual lobe with an origin directly below the observer.

The equations in Appendix D.1 may be solved for R_g and β to define the shape of the lobe which may then be integrated to determine its area. In particular, by setting $\beta = 0$ the equations give the ground range to the far and near points of the lobe and hence the lobe length. Then, the lobe width at any point along its length may be calculated (Appendix D.4).

With these lobe dimensions, it is possible to calculate the lobe area and the probability of seeing the target in the glimpse. However, we will first consider the features of the calculation of lobe area which are peculiar to television, and some complications which have arisen in the calculation of lobe dimensions.

4.2 Television

With television, the calculation of visual lobes is similar to the visual calculations excepting that one has the choice of either calculating lobe sizes on the display or in the ground plane. The difference only comes in during the calculation of cumulative probabilities under the assumption of a uniformly random distribution of glimpsing. In the first case, the glimpsing is assumed uniformly random over the display, and in the second case, uniformly random over the ground plane. The average results from the two calculations are only significantly different with long search areas when the first case would concentrate search more in the distant parts of the search area on the ground plane. Certainly, this difference would not be revealed by the approximations used in present calculations for the average ground lobe size. In Appendix D.5 is shown the details of calculating lobe sizes in the ground plane.

4.3 Complications encountered in the calculation of the ground ranges

In solving the equations which give the ground ranges from the observer to both ends of the visual lobe, certain difficulties have been encountered

due to two causes. Firstly, during the iteration process to determine the range to the far point of a lobe, an intermediate range is sometimes obtained that is greater than the foveal detection range of the target. This situation is merely one of stability in the iterative solution of the equations and introduces no visual difficulties. It is discussed in detail in Appendix D.2. Secondly, it is possible, in certain conditions with glimpse at grazing incidence to the ground, for the visual lobe which is a volume of revolution to intersect the ground at four points, thus giving two separate visual lobe areas. This in turn leads to multiple solutions for the equations giving the range to the near point of the lobe. In this situation, the theory predicts that for an observer looking steadily ahead and flying towards the target, the target first becomes visible, then invisible, and then, at the last moment below him, visible again in the observer's peripheral vision. The size of the near visual lobe can be quite significant and poses the question of whether to include it or leave it out of the effective visual lobe area. Its size is very sensitive to the assumptions regarding the efficacy of peripheral vision in a complex background and in any case, it only picks up targets which are too close to the observer to be of any interest in the present context, so in general it has been ignored in calculations. This does, however, sometimes result in a step function in the calculation of lobe sizes as the lobes change from being a long thin one to two separate ones. With search areas of finite length this often does not matter as the long thin lobes and the rear portion of the double lobe are curtailed by the limits of the search area. This situation together with an example of where it occurs is discussed in detail in Appendix E.

5 THE CALCULATION OF LOBE AREA AND GLIMPSE PROBABILITIES

In section 4, methods have been described which enable the end points of the lobes (and hence length), together with their width at any point along their length to be calculated. The lobe shape is generally nearly elliptical which suggests the approximation of calculating the lobe area directly from its length and width. In the calculation of lobe areas with the elliptic approximation, the width of the lobe is not necessarily taken to be the maximum width, but the width at a point half way along its length. This does give the maximum width for lobes which are elliptical, but gives a better estimate for the area of lobes which are not. As an example of the errors involved in this approximation calculations have been made for selected values of target size and inherent contrast etc. to obtain the shape of the visual lobe together with its exact and equivalent elliptic lobe areas. Fig.11 shows how the shape of the lobe for a particular target varies as the observer

flies towards it. A flat target was chosen as these tend to result in lobes which are rather bulbous towards the observer and are thus a severe test of the elliptic approximation. The lobes associated with the first, third and sixth glimpse are shown when the lobe gradually changes from a pear shape to an ellipse. In Fig.12 is plotted a curve showing how the ratio of elliptic to exact lobe area varies with the range to the search area. At worst, the elliptic approximation over-estimates the true area by about 16% which is considered within the limits of accuracy of the theory. However, it may be seen in Fig.13 that the difference between the cumulative detection probabilities calculated from equation (1) using the exact and elliptic lobe areas is negligible. Thus, for the calculation of subsequent lobes that lie entirely within the search area, the lobes have been considered simply as ellipses, involving the calculation of their width only at the mid point of their length. Larger errors than the 20% found above may occur with difficult targets at low altitudes, but the decreasing accuracy of the approximation under such circumstances is probably well matched by the decreasing accuracy of the visual theory, due to the uncertainties of the efficacy of peripheral vision, terrain screening etc.

5.1 It is readily appreciated from the lengths of the lobes in Fig.11, particularly in their extension towards the observer, that in some circumstances much of the lobe will be outside the search area and hence wasted. The extent of the lobe in the flight direction, and its overlap with the search area is adequately accounted for in the calculation of average probabilities for targets anywhere in the search area by the use of equation (6) in the calculation of glimpse probabilities. It is, however, readily appreciated from Fig.11a that the degree of overlap for the forward and rearward parts of the lobe can be quite different. Consequently, equation (6) needs to be modified to make separate allowance for the overlap at each end of the lobe, and equation (7) below is used in which ℓ_1 and ℓ_2 are the forward and rearward lengths of the equivalent rectangular lobe from the glimpse axis.

$$p = \left[\frac{\ell_1}{X} - \frac{\ell_1^2}{2X^2} + \frac{\ell_2}{X} - \frac{\ell_2^2}{2X^2}\right]\left[\frac{w}{Y} - \frac{w^2}{4Y^2}\right] \qquad (7a)$$

In relation to equation (6), $\ell_1 + \ell_2 = x$, and $w = y$, the total width of the lobe. The search area dimensions X, Y, are unchanged. In relation to the calculated shape of an elliptic lobe, $\ell_1 = \sqrt{\frac{\pi}{4}}\,\ell_1'$, $\ell_2 = \sqrt{\frac{\pi}{4}}\,\ell_2'$ and $w = \sqrt{\frac{\pi}{4}}\,w'$,

where the dashed quantities are the overall dimensions calculated for an elliptic lobe.

The limits of applicability of equation (7a) are as follows.

Consider equation (7a) to be in the form

$$p = [p_1 + p_2]\,[p_3]$$

where $p_1 = \dfrac{\ell_1}{X} - \dfrac{\ell_1^2}{2X^2}$,

$$p_2 = \frac{\ell_2}{X} - \frac{\ell_2^2}{2X^2}$$

and $p_3 = \dfrac{w}{Y} - \dfrac{w^2}{4Y^2}$

$$\left.\begin{array}{lll}
\text{Then, for} & & \\
& \ell_1 < X, & p_1 = \dfrac{\ell_1}{X} - \dfrac{\ell_1^2}{2X^2} \\
& \ell_1 > X, & p_1 = \dfrac{1}{2} \\
\text{Also, for} & & \\
& \ell_2 < X, & p_2 = \dfrac{\ell_2}{X} - \dfrac{\ell_2^2}{2X^2} \\
& \ell_2 > X, & p_2 = \dfrac{1}{2} \\
\text{and for} & & \\
& w < 2Y, & p_3 = \dfrac{w}{Y} - \dfrac{w^2}{4Y^2} \\
& w > Y, & p_3 = 1{\cdot}0
\end{array}\right\} \qquad (7b)$$

Equations (7) account adequately for the amount of lobe wasted outside the search area, but, again with reference to Fig.11a, it is seen that under the normal method of calculating the width of the lobe at the mid length point the lobe width may be very much larger than the useful width of the lobe corresponding to the part inside the search area. In these circumstances the useful width is taken to be the average width of the part of the length of

the lobe inside the search area. Three samples along this length, the far point of the lobe or the far edge of the search area, the near edge of the search area, and a point halfway between, have been used successfully. These lobe dimensions are again converted to those of the equivalent rectangular lobe for use in equations (7).

5.2 In some applications of visual search theory it may be required to determine visual lobe sizes for targets at particular parts of the search area. However, in general an average value for the whole search area is required. If the search area is not too wide, it may be assumed that there is no lateral variation in lobe size and most calculations have simply taken three samples in range along the centre-line of the search area, the near point, the far point and the centre point. As a further approximation, only the lobe size at the centre of the search area has been taken, and in Figs.14a and b the difference between cumulative probability distributions for the three sample and centre point sample method of computing average lobe sizes is shown. The target was assumed to be flat; search area lengths of 1500, 3000 and 6000 ft were considered and two low observer altitudes were assumed. All these factors combine to make a severe test of the adequacy of the centre sample technique and it is seen that the differences between the two methods of calculation increase as the search area length is increased towards the foveal detection range. Increasing the altitude also increases the differences between the two methods. However, these approximations for obtaining an average visual lobe size should be used with great care and each calculation should be considered on its merits.

5.3 With television, or any other optical system of limited field of view, part of the area may not be in the field of view. This does not affect the sizes of visual lobes, which are calculated in a manner similar to the visual calculations. (The equations are detailed in Appendix D.) However, it does affect the choice of sampling points for the calculation of lobe sizes, the calculation of glimpse probabilities through the correction for lobe overlap with the edge of the visual search area, now the edge of the field of view, and the calculation of cumulative probabilities, in that with a steadily decreasing amount of the total search area being covered by the field of view equation (1) no longer applies. These three points are enlarged upon in the following paragraphs.

The choice of sampling points for the calculation of visual lobe sizes depends to some extent on where the camera is assumed to be pointing. If it

always points to the centre of the search area the sampling points would be constrained to be within the field of view, and in the three sample case would be the forward edge, the centre and rearward edge. If, however, the camera is presumed to be directed at some random point of the search area, then the average televisual performance for the whole search area is required and the sampling points would be distributed over it as if there were no field of view restrictions.

The calculation of glimpse probabilities from lobe sizes follows equations (7), but when the field of view is restricted, the values of X and Y used in the equation should be the length and width of this field of view on the ground plane rather than the dimensions of the search area. In practice this is easily done by testing for field of view cut off before computing any lobe sizes and, if lobe overlap occurs, using in equation (7) the field of view limits instead of the whole search area. The way in which the lateral field of view cut off is handled needs further consideration as the lateral ground limits to the field of view are not usually parallel with the sides of the search area.

There are at least eleven ways in which the trapezoidal ground projection of the field of view of a television camera can overlay a rectangular search area, even with the centre of the television field concentric with the search area, (this does not necessarily mean that this is the centre of the trapezoid), and the camera axis aligned with the search area. Fortunately, to date no calculations have been encountered in which the field of view limitation has been of great importance and it has sufficed to make the very crude approximation that lateral field cut off occurs when it occurs across the centre of the field of view. In this case the part of the search area within the field of view has been taken to be $(R_{max} - R_{min}) \times$ width, where the width is that across the centre of the field of view, and R_{max} and R_{min} are the ground fore and aft limits to search determined either by the limits of the search area or the field of view. It is, of course, easy to check whether the fore and aft limits to the field of view are inside or outside the search area, since they are parallel to the fore and aft edges of the search area. In the programmes, it has been a simple matter to check for field restriction prior to any further calculations of lobe sizes etc. and, if field restrictions occur, to substitute the field limits for the search area limits; this automatically accounts for many requirements of the subsequent calculations and the search area limits only have to be resurrected for the calculation of cumulative probabilities according to equation (3).

The third effect of a field of view limitation is that the accumulation of glimpse probabilities to give cumulative probabilities of seeing a target no longer follows the simple form of equation (1). If the television camera approaches the search area such that the part of the search area available for search at any given time is always within that at any previous instant of time, equation (3) may be applied. The use of equation (3) requires calculation of the part of the search area which is common with the field of the television camera. This is considered in the previous paragraph.

6 GLIMPSE TIME

Each separate glimpse of visual search takes a finite time during which vision is fixated on a single point in the search field. During this fixation time, T seconds, the observer in an aircraft will move towards the target by VT ft. Consequently the fixation time, T, is an important factor in determining how many glimpses an observer can make during a fly-past. Unfortunately, there is no clear-cut answer to the correct value of T to be used in the theory. Measurements of T in many different visual tasks result in values around 1/3 second, but past analyses of outdoor search data have favoured values of T as high as $1\frac{1}{2}$ seconds. This factor of five uncertainty in the value of T reflects directly on the predictions of search theory. In Ref.1 it was shown that there were many possible reasons for the increased value of T in outdoor search, some genuinely associated with an increased T, such as clustering of glimpses for the inspection of single points more closely than allowed for by a single glimpse, and some merely of the situation analysed, such as the erroneous estimation of lobe sizes or the inclusion in the experimental distributions of the effects of factors other than pure search. It was also shown that the effect of being optimistic in the estimation of lobe sizes was quite inseparable in experimental data from having to use longer glimpse times. In view of these factors, the appropriate glimpse time for any given situation may well be a function of target difficulty, background complexity and the threshold data used to predict the sizes of visual lobes, and when matching theory with experiment the degree of control over the additional experimental factors could also be decisive in defining a suitable value for T.

In theoretical predictions to date, values for T of 1/3 second have been used with J. H. Taylor's visual lobe sizes for simple search situations in accordance with the experiments analysed in Ref.1. For complex search situations the value of 1·5 seconds has been used but this is under constant review.

7 CONCLUSIONS

This document shows in detail how search theory has been applied to the task of air to ground target detection in order to calculate the probability of detecting a target situated in a search area towards which an observer is flying. A number of features of the calculations have been discussed, including, the treatment of some of the more unusual lobe shapes met with in low altitude conditions, the best way of calculating glimpse probabilities from lobe areas and the glimpse times to be used in the theory. In particular, it has been demonstrated that for most cases it is within the limits of accuracy of the theory to calculate the area of the ground section of the visual lobe assuming it is an ellipse. Also, it has been shown that the average lobe for the whole search area can, in most cases, be taken with sufficient accuracy to be the one associated with a glimpse to the centre of the search area. Computer programmes have been written to expedite the calculation of cumulative detection probability and programmes are available to deal with the cases of flight at a constant dive angle on to the centre of the search area and flight at constant altitude, both for visual and televisual viewing.

The estimation of visual performance, as represented by the present calculations, is the subject of constant review, both in terms of the fundamental concepts involved and the numerical data used. Some favourable comparisons with practice are found, with the review of the theory, in Ref.1 for some simple search situations. Comparisons with actual flight trials are also continuing, but the proportion of flight data which is amenable to detailed comparison with theory is small as the measurement of all the necessary factors in the flight trials would involve a prohibitively large work load. However, many facets of the R.A.E. flying programme are being improved to allow for better interpretation in terms of theoretical concepts.

Appendix A

A PROBLEM OF CUMULATIVE PROBABILITY OCCURRING DURING VISUAL SEARCH

by A. R. Runnalls

A.1 Introduction

Visual search is presumed to take place in a series of glimpses, each glimpse taking in a limited part of the search area. In this Report both the distribution of glimpsing and the target distribution are assumed to be uniformly random within their respective limits.

The probability of seeing a target within the search area in a single glimpse is simply a/A, where a is the area covered by the eye in a single glimpse - the visual lobe area - and A is the search area. The probability of seeing such a target after n similar glimpses will then be:-

$$p = 1 - \left(1 - \frac{a}{A}\right)^n .$$

However, with the use of optical devices of limited field of view, it often happens that the search area A is smaller than the area of target distribution Q. If in the above the target is within an area $Q \geqslant A$ the probability of seeing it becomes simply A/Q times p above. More generally A may vary, and in particular when the observer and optical system are approaching the target area, simple modifications of the above formula do not suffice and a more involved consideration of the interdependence of the variables is needed.

This Appendix derives from elementary principles the correct formula in this case, where the field of view available to an observer at each glimpse is less than the total search area in which the target may lie, and decreases steadily with time. Each field at a given instant is assumed to be wholly within the field at a previous instant. The observer is assumed to search uniformly randomly all over the part of the search area available to him in his field of view.

A.2 Theory

A.2.1 Notation

Q is the total area within which the target is (uniformly) distributed

A_n is the area within Q which is available to search at the nth glimpse

a_n is the area covered by the nth glimpse

Z_n denotes the event that the target lies within the search area at the nth glimpse

S_n denotes the event that the target is seen in the nth glimpse

σ_n denotes the event that the target is seen in at least one glimpse up to and including the nth, i.e.

$$\sigma_n = \bigcup_{r=1}^{n} (S_r)$$

$p(\sigma_o)$ will denote the a priori probability of having seen the target, being, for this calculation, nil

$\in$ is a member of

$\bigcup_{i=1}^{n} A_i$ all x such that $x \in A_i$ for at least one integer i, $1 \leq i \leq n$

$\bigcap_{i=1}^{n} A_i$ all x such that $x \in A_i$ for each integer i, $1 \leq i \leq n$

$\supset$ contains

$|$ restricted to

$\sim$ negative event

$\prod_{i=1}^{n} A_i$ direct product, $A_1 A_2 A_3 \ldots . A_n$

A.2.2 Calculation

We have, as the basis for subsequent mathematics the following axioms

I $\quad p(\sim \sigma_n) = p(\sim \sigma_{n-1} \cdot \sim s_n)$

II $\quad p(Z_n) = \dfrac{A_n}{Q}$

III $\quad p(S_n | Z_n) = \dfrac{a_n}{A_n}$

IV $$p(S_n | \sim Z_n) = 0$$

V $$Z_n \supset \bigcap_{r=1}^{n} (Z_r) \supset \bigcap_{r=1}^{n-1} (Z_r)$$

We have, by II and III:-

$$p(S_n | Z_n) = \frac{p(S_n . Z_n)}{p(Z_n)} = \frac{a_n}{A_n} \tag{A-1}$$

therefore

$$p(S_n . Z_n) = \frac{a_n \, p(Z_n)}{A_n} = \frac{a_n}{Q}$$

Similarly by II and IV

$$p(S_n . \sim Z_n) = 0$$

therefore

$$p(S_n) = p(S_n . Z_n) + p(S_n . \sim Z_n) = \frac{a_n}{Q} \tag{A-2}$$

Now

$$p(\sim \sigma_{n-1}) = p(\sim \sigma_{n-1} . S_n) + p(\sim \sigma_{n-1} . \sim S_n) \tag{A-3}$$

and

$$p(\sim \sigma_{n-1} . S_n) = p(\sim \sigma_{n-1} | S_n) \times p(S_n) \tag{A-4}$$

or in the trivial case n = 1

$$p(\sim \sigma_o . S_1) = p(S_1) \quad \text{since} \quad p(\sim \sigma_o) = 1 \tag{A-4a}$$

By IV and V

$$S_n \supset Z_n \supset \bigcap_{r=1}^{n-1} (Z_r) \tag{A-5}$$

By III

$$p(\sim S_n | Z_n) = 1 - \frac{a_n}{A_n} \tag{A-6}$$

therefore

$$p\left[\bigcap_{r=1}^{n-1}(\sim S_r) \,\middle|\, \bigcap_{r=1}^{n-1}(Z_r)\right] = \prod_{r=1}^{n-1}\left(1 - \frac{a_r}{A_r}\right)$$

since these events $\sim S_r | Z_r$ are clearly independent.

Also, by definition

$$p\left[\bigcap_{r=1}^{n-1}(\sim S_r) \,\middle|\, \bigcap_{r=1}^{n-1}(Z_r)\right] = p\left[\sim \sigma_{n-1} \,\middle|\, \bigcap_{r=1}^{n-1}(Z_r)\right]$$

By (A-5)

$$S_n \supset \bigcap_{r=1}^{n-1}(Z_r) \tag{A-7}$$

therefore

$$p\left[\sim \sigma_{n-1} \,\middle|\, \bigcap_{r=1}^{n-1}(Z_r)\right] = p(\sim \sigma_{n-1} | S_n)$$

since it is only because both S_n and σ_{n-1} depend on the events Z_r that they are not independent.

Hence

$$p(\sim \sigma_{n-1} | S_n) = \prod_{r=1}^{n-1}\left(1 - \frac{a_r}{A_r}\right)$$

By (A-4)

$$p(\sim \sigma_{n-1} \cdot S_n) = p(S_n)\prod_{r=1}^{n-1}\left(1 - \frac{a_r}{A_r}\right) \tag{A-8}$$

or

$$p(\sim \sigma_o \cdot S_1) = p(S_1) \quad \text{if} \quad n = 1 \tag{A-8a}$$

By (A-3)

$$p(\sim \sigma_{n-1} \cdot \sim S_n) = p(\sim \sigma_{n-1}) - p(\sim \sigma_{n-1} \cdot S_n) \tag{A-9}$$

i.e.

$$p(\sim \sigma_n) = p(\sim \sigma_{n-1}) - p(S_n) \prod_{r=1}^{n-1} \left(1 - \frac{a_r}{A_r}\right) \qquad \text{(A-9a)}$$

or if n = 1.

$$p(\sim \sigma_1) = p(\sim \sigma_o) - p(S_1) = 1 - p(S_1)$$

This difference equation (9) is probably the most convenient computational form for the required cumulative glimpse probability $p(\sigma_n)$. It may be rewritten as

$$p(\sigma_n) = p(\sigma_{n-1}) + \frac{a_n}{Q} \prod_{r=1}^{n-1} \left(1 - \frac{a_r}{A_r}\right)$$

Of course, the glimpse probabilities a_r/A_r have not been corrected for the effects of edges, and a more complete form of the equation is found in the main text as equation (3) coupled with equation (7). However, continuing to the determination of $p(\sigma_n)$,

$$p(\sim \sigma_n) = p(\sim \sigma_1) - \sum_{k=2}^{n} \left[p(S_n) \prod_{r=1}^{k-1} \left(1 - \frac{a_r}{A_r}\right) \right]$$

$$= 1 - p(S_1) - \sum_{k=2}^{n} \left[p(S_n) \prod_{r=1}^{k-1} \left(1 - \frac{a_r}{A_r}\right) \right]$$

$$= 1 - \frac{a_1}{Q} - \sum_{k=2}^{n} \left[\frac{a_k}{Q} \prod_{r=1}^{k-1} \left(1 - \frac{a_r}{A_r}\right) \right] \qquad \text{(A-10)}$$

i.e.

$$p(\sigma_n) = \frac{a_1}{Q} + \sum_{k=2}^{n} \left[\frac{a_k}{Q} \prod_{r=1}^{k-1} \left(1 - \frac{a_r}{A_r}\right) \right]$$

this being the required formula, valid when $n \geqslant 2$.

A.3 Conclusion

An equation has been derived for the cumulative probability of seeing a target when the part of the total target distribution area actually available for visual search is progressively reduced.

Appendix B

CALCULATION OF GLIMPSE PROBABILITY

The probability of detecting the target in a single glimpse to the search area, p_g, can be calculated from the ratio:-

$$p_g = \text{lobe area associated with glimpse/search area}$$

i.e.

$$p_g = \frac{a}{A} \tag{B-1}$$

However, this simple expression makes no allowance for glimpses directed to the edges of the search area where the lobes extend outside and consequently does not asymptote to the correct limit of unity when the lobe areas become larger than the search area. To allow for this situation, two different methods have been considered.

The popular method of dealing with this is simply to use

$$p_g = \frac{a}{(A + a)} \tag{B-2}$$

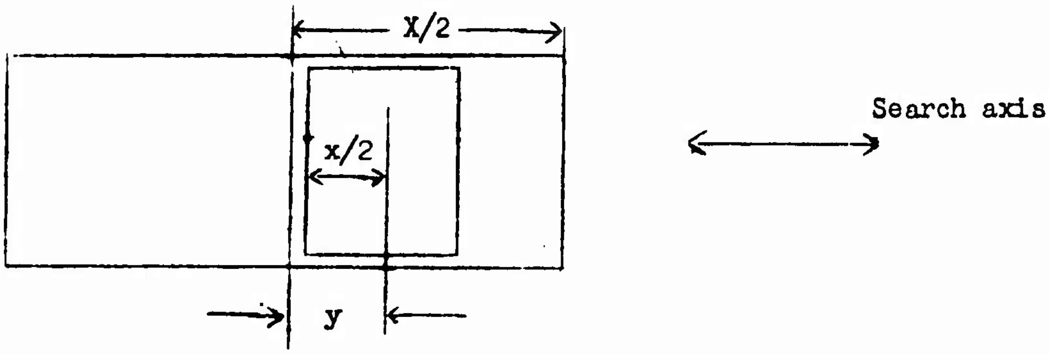
This form of the correction is not a good one and should be replaced with one derived in Ref.1. The derivation is primarily an extension of the exact solution for search along a line, which is illustrated below.

Consider the search along a line of length X with a visual lobe of length x.

Three separate situations occur in this analysis, (i) when $x < X$, (ii) when $X < x < 2X$, and (iii) when $x > 2X$. These are analysed separately in the following sections.

(i) Lobe length smaller than search length

Search area, length X, with lobe length x, $x < X$. Glimpses fall uniformly randomly in the length X,



The useful lobe length within the search area for a glimpse to a point distance y from the centre of the search area can be expressed by the following equations

$$0 < y < \frac{X}{2} - \frac{x}{2}, \qquad \text{useful lobe length} = x$$

$$\frac{X}{2} - \frac{x}{2} < y < \frac{X}{2}, \qquad \text{useful lobe length} = \frac{x}{2} + \frac{X}{2} - y$$

The average useful length for glimpses from $0 < y < X/2$ is

$$\frac{1}{X/2}\left\{\int_0^{X/2-x/2} x\,dy + \int_{X/2-x/2}^{X/2}\left[\frac{x}{2} + \frac{X}{2} - y\right]dy\right\} = \left[\frac{x}{2} - \frac{x^2}{8X}\right]\frac{X}{X/2} \qquad \text{(B-3)}$$

For $-X/2 < y < 0$, a similar expression is obtained so that this expression gives the average lobe length for glimpses falling anywhere in the length of the search area,

i.e.

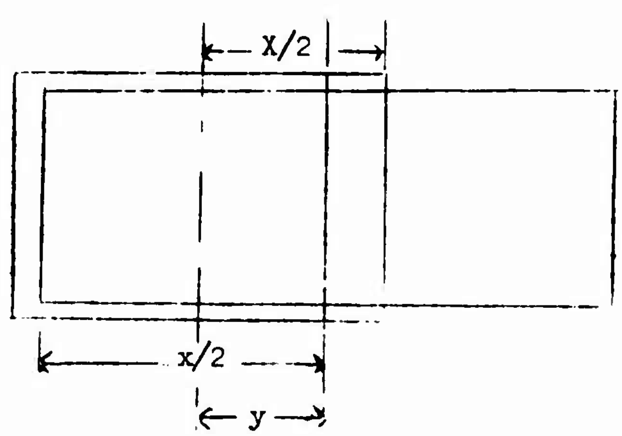
$$\left[x - \frac{x^2}{4X}\right]$$

The average probability of seeing the target for glimpses directed anywhere in the search length is

$$p_g = \left[x - \frac{x^2}{4X}\right]\Big/X \qquad \text{(B-4)}$$

(ii) <u>Lobe length longer than the search length</u>

i.e. For $2X > x > X$



The useful lobe length within the search area for a glimpse to a point distance y from the centre of the search area can be expressed by the following equations.

$$0 < y < \frac{x}{2} - \frac{X}{2}, \qquad \text{useful lobe length} = X$$

$$\frac{x}{2} - \frac{X}{2} < y < \frac{X}{2}, \qquad \text{useful lobe length} = \frac{X}{2} + \frac{x}{2} - y$$

Therefore, the average useful length for glimpses from $0 < y < \frac{X}{2}$

$$= \frac{1}{X/2}\left\{\int_0^{x/2-X/2} X\,dy + \int_{x/2-X/2}^{x/2}\left[\frac{X}{2} + \frac{x}{2} - y\right] dy\right\} = \left[x - \frac{x^2}{4X}\right]$$

For $-X/2 < y < 0$, a similar expression is obtained, hence as for (i) the probability of seeing the target is

$$p_g = \left[x - \frac{x^2}{4X}\right]\Big/X \qquad \text{(B-5)}$$

This is the same as for (i) so that a single expression covers both cases.

(iii) <u>For $x > 2X, p_g = 1{\cdot}0$ at all times</u>

This is not the same as the equations for cases (i) and (ii) so that $x = 2X$ with $p_g = 1{\cdot}0$ is a definite cut off to the validity of equation (B-5) above.

Equations (B-4) or (B-5) thus provide an exact solution to the problem of allowing for lobe overlap with the edge of a search area for search along a single axis. In Fig.1, this exact solution is compared with the one-dimensional equivalent of equation (B-2) viz. $p = x/(X + x)$. It is seen that the exact solution asymptotes to unity much quicker than the solution usually used, and that the latter is quite pessimistic in predicting large glimpse probabilities.

Equation (B-4) needs to be generalised for the interaction of general lobe shapes with general search area proportions. For the present, the equation

$$p = \left(\frac{x}{X} - \frac{x^2}{4X^2}\right)\left(\frac{y}{Y} - \frac{y^2}{4Y^2}\right) \qquad \text{(B-6)}$$

which is reproduced in the main text, with a form applicable to symmetrical lobes and search areas, as equations (6) is used. This considers each axis of search as separate and uses equation (B-4) for each axis. This composite equation has the following desirable properties.

(i) It is exact for lobes which are long and thin compared with the search area.

(ii) It is a correct representation of the following two limits for either square lobes in a square search area or circular areas in a circular search area.

(a) $p_g \rightarrow a/A$ for small a, where a is the lobe area and A the search area.

(b) $p_g = 1{\cdot}0$ for $a = 4A$.

The usual formula used only satisfies (ii)(a), and $p_g = 1{\cdot}0$ but for the wrong limiting value of a/A.

A form of equation (B-6) which is used with lobes of different nose and tail lengths is found in the main text as equations (7).

Appendix C
(see section 3)

CALCULATION OF FOVEAL DETECTION RANGE, F, AT THE START OF A RUN

C.1 Visual situation

The calculation of these ranges is straightforward and has been presented in Ref.3, but some features of the technique used at present are different and so a brief outline is given here.

The apparent contrast available at the observer's eye, C_R at range R from the target is given by

$$C_R = \frac{C_o}{\{1 + b[\exp(\sigma R) - 1]\}} \qquad \text{(C-1)}$$

where C_o is the target inherent contrast,

b is the sky/ground luminance ratio, and

σ is the atmospheric attenuation coefficient.

The relations between range and angle subtended by the target at the eye are:-

For a target normal to the sight-line (either dive or horizontal approach)

$$R = \left(\frac{10800}{\pi}\right)\left(\frac{d_o}{\alpha}\right) \qquad \text{(C-2)}$$

and for a flat target (horizontal approach)

$$R = \left[\frac{10800}{\pi} \left(\frac{d_o}{\alpha}\right) (H)^{0\cdot5}\right]^{0\cdot667} \qquad \text{(C-3)}$$

where d_o is the equivalent diameter of the target,

α is the angle subtended by the target at the observer's eye, and

H is the altitude of the observer.

The eye's foveal detection performance is defined by the Blackwell 8 position search in 6 seconds threshold data shifted along the log (contrast) axis by a degradation constant, see Fig.3. This curve is a plot of log (target threshold contrast) - v - log (target visual angle). For the first stage of this calculation, atmospheric attenuation is neglected and an initial value of target

visual angle is found from the threshold curve by setting target apparent contrast equal to target inherent contrast, and a range corresponding to this visual angle obtained from equations (C-2) or (C-3) and stored as R_1, while the apparent contrast C_R is calculated from (C-1).

The above set of calculations is repeated by entering the threshold curve with the latest value of $(\log C_R)$ to determine α and hence a new range R_2. At this stage the modulus of $(R_2/R_1 - 1)$ is obtained and if it is smaller than a small quantity, say 0·01, the foveal detection range, F, is set equal to R_2. If this condition is not satisfied, the loop is repeated to find a new R_2 until the modulus is satisfied.

C.2 Television detection

As a simple extension of the visual case, the equations for the detection of a target through television, or any other similar optical system, are as follows. A $\frac{1}{2}$ inch scanned width on the television faceplate is assumed, but for any other scanned width the geometry of equations (C-8) or (C-9) below are simply scaled accordingly.

The contrast at the observer's eye is given by

$$C = \gamma C_R F(\nu) \tag{C-4}$$

where γ is the contrast enhancement of the television chain, C_R is the contrast of the target at the lens of the camera and $F(\nu)$ is the frequency response curve of the camera as a function of ν, a factor describing the size of the target in terms of the size of the television scan. [ν is, in fact, defined by

$$\nu = \frac{\text{width of the television scan}}{\text{width of the target on the scan}} ; \tag{C-5}$$

it is often known by that misleading term, the 'line number'; misleading, as it has nothing whatsoever to do with the 'line standard' of the television system.]

The frequency response curves used in calculations are shown in Fig.4. These are estimates from genuine response curves both to allow for the fact that targets are usually more square than the long thin lines typical of a television test grid, and the fact that the longer tail of a genuine television horizontal response curve is neither matched by the vertical response nor of

any use visually. Each curve in Fig.4 is labelled by a maximum value of ν, ν_{max}, which is effectively the limit of vision. The value of ν_{max} is taken at $F(\nu) = 0{\cdot}05$, but as the curves used are so steep in this region this value of ν is not much different from that at $F(\nu) = 0$.

The contrast at the television lens is given by

$$C_R = \frac{C_o}{\{1 + b[\exp(\sigma R) - 1]\}} \qquad (C\text{-}6)$$

as for the visual case.

The target subtense at the eye is given by

$$\alpha = 60 \frac{\Omega}{\nu} \quad \text{min of arc} \qquad (C\text{-}7)$$

where Ω is the angle, in degrees, subtended by the whole display at the eye. The range corresponding to this value of α is

$$R = 120\, \Omega\, f \frac{d_o}{\alpha} \qquad (C\text{-}8)$$

for a target of effective diameter d_o which is always normal to the line of sight, or

$$R = \left[\frac{120\, \Omega\, f\, d_o\, (H)^{0{\cdot}5}}{\alpha}\right]^{0{\cdot}667} \qquad (C\text{-}9)$$

for a similar target which is lying flat on the ground. H is the altitude of the observer in feet.

At the foveal detection range $R = F$, $C = \varepsilon_o$, the foveal threshold derived from the threshold curve as a function of α and any other factors which may be considered to affect visual thresholds in the circumstance analysed.

The calculation of a detection range is started by determining an initial value of α from equation (C-7). The corresponding range is calculated from equation (C-8) or (C-9), together with the contrast on the screen from (C-4) and (C-6). The threshold curve is interpolated with this contrast to find a new log α, α, ν, $F(\nu)$ etc. The new range, R_2, calculated is tested against the previous value for R for $|R_2/R_1 - 1| > 0{\cdot}01$, and the iterative loop is repeated, if necessary, to give finally a value for F, equal to R_2.

It frequently happened in these iterations that a stable state of oscillation about the range corresponding to ν_{max} was reached. This was avoided by testing all new estimates of detection range against the maximum possible range, R_{max}, corresponding to ν_{max}, and if they were larger they were replaced by R_{max}.

This calculation provides a value of range from which to start the search calculations.

Appendix D

CALCULATION OF THE SHAPE AND SIZE OF A VISUAL LOBE

Visual situation

D.1 Equations defining the general point on a ground section lobe

Consider a glimpse at the point A in Fig.5, and the general point C on the ground lobe section about A. In Fig.5, θ is the allowable off axis angle for vision for the particular glimpse situation considered, φ is the depression angle of the glimpse, and β is simply the ground projection of θ.

Applying the cosine rule to Δ's ABC, AEC and equating for the common side AC gives

$$AC^2 = AB^2 + BC^2 - 2AB\ BC \cos A\hat{B}C = AE^2 + CE^2 - 2AE\ CE \cos A\hat{E}C \quad \text{(D-1)}$$

i.e.

$$d^2 + R_s^2 - 2d\,R_s \cos\theta = (H\cot\varphi)^2 + R_g^2 - 2H\,R_g \cot\varphi\cos\beta \quad \text{(D-2)}$$

Substituting,

$$d = H \operatorname{cosec}\varphi$$

and

$$R_s = (R_g^2 + H^2)^{0\cdot5}$$

in equation (D-2) and rearranging gives,

$$R_g \cos\beta = (R_g^2 + H^2)^{0\cdot5}\frac{\cos\theta}{\cos\varphi} - H\tan\varphi \quad \text{(D-3)}$$

In particular, the ground co-ordinates of the far and near points of the lobe, for which $\beta = 0$, and are given by

$$R_g = H\cot[\varphi \pm \theta] \quad \text{(D-4)}$$

For the far point of the lobe, the solution giving the maximum range is required and hence the negative sign is taken, and for the near point of the lobe the position sign is taken. Since θ itself is a function of R_g, this equation can only be solved iteratively and the two solutions are treated separately below.

D.2 Ground range to far point

Initially the slant range to the far point of the lobe, Y, is calculated from

$$Y = (E^2 + H^2)^{0.5} \qquad \text{(D-5)}$$

where E is the ground range from the observer to the centre of the glimpse, which is initially set to F, the foveal detection ground detection range. The existence of H makes Y a little larger than the foveal range which ensures that the search calculations start just beyond the foveal range. From this point, the calculation is iterative. It starts with the calculation of the target visual angle α from either

(a) for target normal to sight-line

$$\alpha = \left(\frac{10800}{\pi}\right)\left(\frac{d_o}{Y}\right) \quad \text{min of arc} \qquad \text{(D-6)}$$

or,

(b) for a flat target,

$$\alpha = \left(\frac{10800}{\pi}\right)\left(\frac{d_o}{Y}\right)\left(\frac{H}{Y}\right)^{0.5} \quad \text{min of arc} \qquad \text{(D-7)}$$

From tabulated logarithmic values of foveal threshold contrast (ε_o) - v - target visual angle (α) the value of $\log \varepsilon_o$ corresponding to the above value of α is calculated. No account is taken of the aspect ratio of the apparent outline of the target.

The target apparent contrast available at the eye is calculated from

$$C_R = \frac{C_o}{\{1 + b[\exp(\sigma R) - 1]\}} \qquad \text{(D-8)}$$

and hence is obtained $\log(C_R/\varepsilon_o)$.

If $\log(C_R/\varepsilon_o) < 0$, the initial range calculated or otherwise inserted at the start of the run is too large and the range E is reset to

$$E - VT$$

where V is the aircraft speed,

and T is the glimpse time.

The slant range Y and $\log(C_R/\varepsilon_o)$ are recalculated.

J. H. Taylor's off-axis data for the eye is given in Fig.2. If $\log(C_R/\varepsilon_o) > 0$, θ is calculated according to a two-straightline approximation to J. H. Taylor's data, viz:

$$\left.\begin{array}{lll} 0 < \log\left(\frac{C_R}{\varepsilon_o}\right) < \log\left(\frac{\varepsilon_4}{\varepsilon_o}\right) & \theta = & 4\log\left(\frac{C_R}{\varepsilon_o}\right) \Big/ \log\left(\frac{\varepsilon_4}{\varepsilon_o}\right) \\ \log\left(\frac{C_R}{\varepsilon_o}\right) > \log\left(\frac{\varepsilon_4}{\varepsilon_o}\right) & \theta = & 4 + 21\cdot6\left[\log\left(\frac{C_R}{\varepsilon_o}\right) - \log\left(\frac{\varepsilon_4}{\varepsilon_o}\right)\right] \\ \text{or} & \theta = & 4 + 9\cdot4\left[\ln\left(\frac{C_R}{\varepsilon_o}\right) - \ln\left(\frac{\varepsilon_4}{\varepsilon_o}\right)\right] \end{array}\right\} \quad \text{(D-9)}$$

where ε_4 is the extra contrast required for vision at 4° off axis, the break point of the two straight lines in the approximation. This is tabulated in the computer with ε_o as a function of $\log\,\alpha$. A degradation factor, g, is introduced in the calculations to allow for the possibility that practical visual lobes are less than those predicted by J. H. Taylor's data, i.e.

$$\theta \text{ becomes } \frac{\theta}{g} \quad \text{(D-10)}$$

Using this value of θ, an initial ground range R_o is calculated from equation (D-4), taking the negative sign. The slant range corresponding to R_o is obtained from equation (D-5) with R_o replacing E, and then are obtained α, $\log(C_R/\varepsilon_o)$ and the next value of θ, θ_1 and hence R_1. When the next iteration has been performed and R_2 obtained, it is possible to use a convergence forcing formula known as Wegstein's iterative method[4] which uses the last 3 iterative values of range in order to calculate a better approximation to the current range. If at the nth iteration the ground range is R_n, then a better approximation $\bar{R}_n$ is calculated from

$$\bar{R}_n = R_n - \frac{(R_n - R_{(n-1)})\,(R_n - \bar{R}_{(n-1)})}{(R_n - R_{(n-1)} - \bar{R}_{(n-1)} + \bar{R}_{(n-2)})} \quad \text{(D-11)}$$

Each value of $\bar{R}_n$ is tested for $|\bar{R}_n/\bar{R}_{(n-1)} - 1| > 0\cdot005$. If the modulus is less than $0\cdot005$, the required range to the lobe far point (R_1) (see Fig.7) is set equal to $\bar{R}_n$, but if greater than $0\cdot005$ then the slant range $Y = (\bar{R}_n^2 + H^2)^{0\cdot5}$ is fed into the iterative loop again at equations (D-6) or (D-7).

Only one complication has arisen in the calculation of the ground range to the far point. At a range not far from the start of a run, during the iterations for the range to the far point of a lobe, R_n, it sometimes happens that the next approximation to the far point ground range calculated from equation (B-11), $\bar{R}_n$, is greater than the foveal detection range, F, possible under the particular viewing conditions. If this occurs, a new value of $\bar{R}_n$ is calculated from:-

$$(F + \bar{R}_{n-1})/2 \tag{D-12}$$

when the iterative process can proceed to obtain the horizontal range to the far point of the lobe, R_1.

D.3 Ground range to near point

A first approximation to the ground range to the near point of the lobe is calculated by assuming that the near point is the same distance behind the glimpse point, (the point in the lobe corresponding to the visual axis), as the far point is ahead of the glimpse point, i.e.

$$R_2 = E - (R_1 - E) \tag{D-13}$$

$$R_2 = 2E - R_1$$

and

$$Y = (R_2^2 + H^2)^{0.5}$$

The method then follows the same pattern as that used for calculating the far point except that equation (D-4) is used with the positive sign to obtain the ground range at each iteration.

D.4 Lobe width

Once the co-ordinates of the lobe end points have been established, the lobe width can be computed at any point D along its length, see Fig.5. Let the width at point D, distance R from E, be W. Considering the common side DC of triangles, ACD and CDE,

$$DC^2 = AC^2 - AD^2 = CE^2 - DE^2 = (BC^2 - BE^2) - (AE - AD)^2$$

i.e.

$$DC^2 = BC^2 - BE^2 - AE^2 + 2AE\ AD - AD^2 \tag{D-14}$$

Also, from triangle ABC,

$$AC^2 = AB^2 + BC^2 - 2AB\ BC \cos\theta \tag{D-15}$$

hence,

$$AB^2 + BC^2 - 2AB\ BC \cos\theta = BC^2 - BE^2 - AE^2 + 2AE\ AD \tag{D-16}$$

writing

$$AB = E \sec\varphi$$

$$BE = H = E \tan\varphi$$

$$AE = E$$

and substituting in equation (D-16)

$$E^2 \sec^2\varphi - 2EY \sec\varphi \cos\theta = -E^2 \tan^2\varphi - E^2 + 2E\,(E-R) \tag{D-17}$$

which gives

$$Y = [E/\cos\varphi - (E-R)\cos\varphi] \sec\theta \tag{D-18}$$

At a particular point along a run, the depression angle of the glimpse, φ and the range to the centre of the glimpse, E are known, while both θ and Y at the range R are unknown. A first approximation to Y is found from

$$Y = (R^2 + H^2)^{0.5}$$

and the corresponding value of θ calculated from equations (D-6) to (D-9). This value of θ is then substituted into equation (D-18) to give another approximation to Y. This iteration for θ and Y is repeated until repeated values of Y do not differ by more than ½%, then this latest value of Y is taken for the slant range to the edge of the lobe at which the width is required. In practice, convergence is usually quite rapid.

Finally,

$$W^2 = Y^2 - H^2 - R^2 \tag{D-19}$$

If W^2 turns out to be negative, it means that there is no lobe at this range. It is often found, particularly with flat targets that two separate visual lobes exist and the point giving negative values for W^2 is between the lobes. This feature warrants further consideration, and in Appendix E ahead it is analysed with some illustrative examples. The simplest, and probably the most

accurate way of handling these double lobes is, for the reasons given in section 5 of the main text, to ignore the rearward lobe.

Televisual situation

D.5 Equations introduced by television

When the actual size of the search area has been established, the co-ordinates of the lobe associated with the glimpse to the centre of the search area are calculated. Wegstein's iterative method is used, as for the visual case, but now the angle (α) subtended at the eye by the image of the target on the monitor is calculated from:-

$$\alpha = \left(\frac{120\ \Omega\ f\ d_0}{Y} \right) \tag{D-20}$$

for a target normal to the sight-line, and

$$\alpha = \frac{120\ \Omega\ f\ d_0\ H^{0\cdot5}}{Y^{1\cdot5}} \tag{D-21}$$

for a flat target.

The line number,

$$\nu = 60\,\frac{\Omega}{\alpha}$$

and the contrast transmitted by the camera $F(\nu)$, at this value of ν is obtained by interpolation in the frequency response curve, Fig.4. The contrast available on the monitor, C_Y, for a slant range Y is given by

$$C_Y = \frac{\gamma\ F(\nu)\ C_0}{\{1 + b[\exp(\sigma Y) - 1]\}} \tag{D-22}$$

The foveal contrast ε_0, and the 4° off-axis contrast ε_4, at this value of α are determined to enable the off-axis angle θ to be calculated. The calculation then follows the method applied to the visual case for the first iteration while equations (D-20) to (D-22) are used again in each subsequent iteration.

Appendix E

COMPLICATIONS ENCOUNTERED IN THE CALCULATION OF THE REARWARD EXTENT OF A VISUAL LOBE

In some search situations, particularly with glimpses of grazing incidence to the search plane as may be obtained with low altitude flight, the problom of multiple solutions to the equations defining the near point of the lobe has been encountered. In these situations the visual lobe was found to consist of two separate portions, one around the axis of vision, and one much closer to the observer, 'beneath him' as it were. This was revealed in the general calculations by a negative $(\text{width})^2$ for the lobe being obtained when a length for the lobe had been established by the computer with no apparent difficulty. These two lobes would suggest that a target coming in from a distance would, with the observer looking constantly at a point ahead, first become visible, then invisible, and then visible again shortly before it passes below the observer. This theoretical anomaly is probably a true reflection of fact, but what to do with this rear lobe is not so obvious. The size of the rear lobe is extremely sensitive to the data assumed to define the peripheral performance of the eye, and so is very suspect when complex terrain backgrounds are encountered and the peripheral data is that of J. H. Taylor, or any other to date, which are all for a plain background. Fortunately, in most cases of interest, detections by these lobes are either too late to be of any practical interest, or would be outside the search area. Consequently, there is every reason to ignore the existence of the rear lobe. If two solutions for the near point exist, the computer programme that has been written to facilitate the calculations obtains the larger solution for the near point range, and the rear lobe is sacrificed as being invalid in normal circumstances.

Referring to Fig.7, the equations to be satisfied simultaneously are:-

$$H = Y_2 \sin(\varphi + \theta_2) \qquad \text{(E-1)}$$

$$R_2 = Y_2 \cos(\varphi + \theta_2) \qquad \text{(E-2)}$$

i.e.

$$\sin(\varphi + \theta_2) = \frac{H}{Y_2} \qquad \text{(E-3)}$$

$$\theta_2 = \sin^{-1}\left(\frac{H}{Y_2}\right) - \varphi \qquad \text{(E-4)}$$

It is convenient to define a function $f(Y)$ depending on the slant range Y from the observer to a point on the lobe near the near point where

$$f(Y) = \sin(\varphi + \theta) - \frac{H}{Y} \qquad \text{(E-5)}$$

so that when $Y = Y_2$ and $\theta = \theta_2$, $f(Y) = 0$.

As an example of where multiple solutions occur, consider the case of an observer in an aircraft approaching and overflying a search area at an altitude of 1000 ft, when the following meteorological and target conditions apply:-

Meteorological visibility,	V =	12 nm ($\sigma = 3 \cdot 0/V = 0 \cdot 25$)
Sky/ground ratio,	b =	2·0
Equivalent diameter of flat target,	d_o =	11·0 ft
Target inherent contrast,	C_o =	1·2

At a horizontal range E = 6750 ft between the observer and centre of the search area, the depression angle φ

$$= \tan^{-1}(1000/6750)$$

i.e

$$\underline{\varphi = 0 \cdot 42^\circ}$$

The solution for the range to the near point of the lobe is the value of Y_2 and the corresponding θ_2 which, when substituted in equation (E-5) makes $f(Y)$ zero. Fig.8 shows $f(Y)$ plotted against Y for values of Y from 2000 ft to 6000 ft where it is seen that there are 3 values of Y_2, i.e. 2290, 3740 and 5500 ft which make $f(Y)$ zero. The off-axis angle, $\theta_{contrast}$ (θ_C) is calculated as a function of Y and plotted in Fig.9. This is the maximum off-axis angle possible at a given range Y. Also plotted in Fig.9 is the geometric off-axis angle (θ_g) obtained from equation (E-4). These two curves, θ_C and θ_g intersect at the 3 values of Y_2 stated above. The explanation for this behaviour is that two separate lobes exist. The first lobe nearer the observer has a slant range to the near point of 2290 ft, and to the far point of 3740 ft, while the slant range to the near point of the second lobe is 5500 ft. A lobe exists for ranges which make $f(Y)$ positive and $\theta_C > \theta_g$. Thus, no lobe exists for ranges between 3740 and 5500 ft, so that the expression for the square of the width is negative in this region, (see Appendix D.4).

When the observer is only 5560 ft from the centre of the search area, where $\varphi = 10{\cdot}2$ degrees, it can be seen from Figs.8 and 9 that there is only one solution to the slant range, approximately 2050 ft that satisfies $f(Y) = 0$ and makes $\theta_C = \theta_g$. A single lobe only, exists for this situation.

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No.	Author	Title, etc.
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2	J. H. Taylor	Private communication in Asbjorn Linge, Visual detection from aircraft. General Dynamics/Convair Engineering Research Rept. ASTIA 270630, p. 11548 (1961)
3	L. J. Smith	Theoretical visual and televisual detection ranges based on target size and contrast. R.A.E. Technical Report 66157 (1966)
4	G. N. Lance	Numerical methods for high speed computers p.134. London, Iliffe & Sons Ltd. (1960)

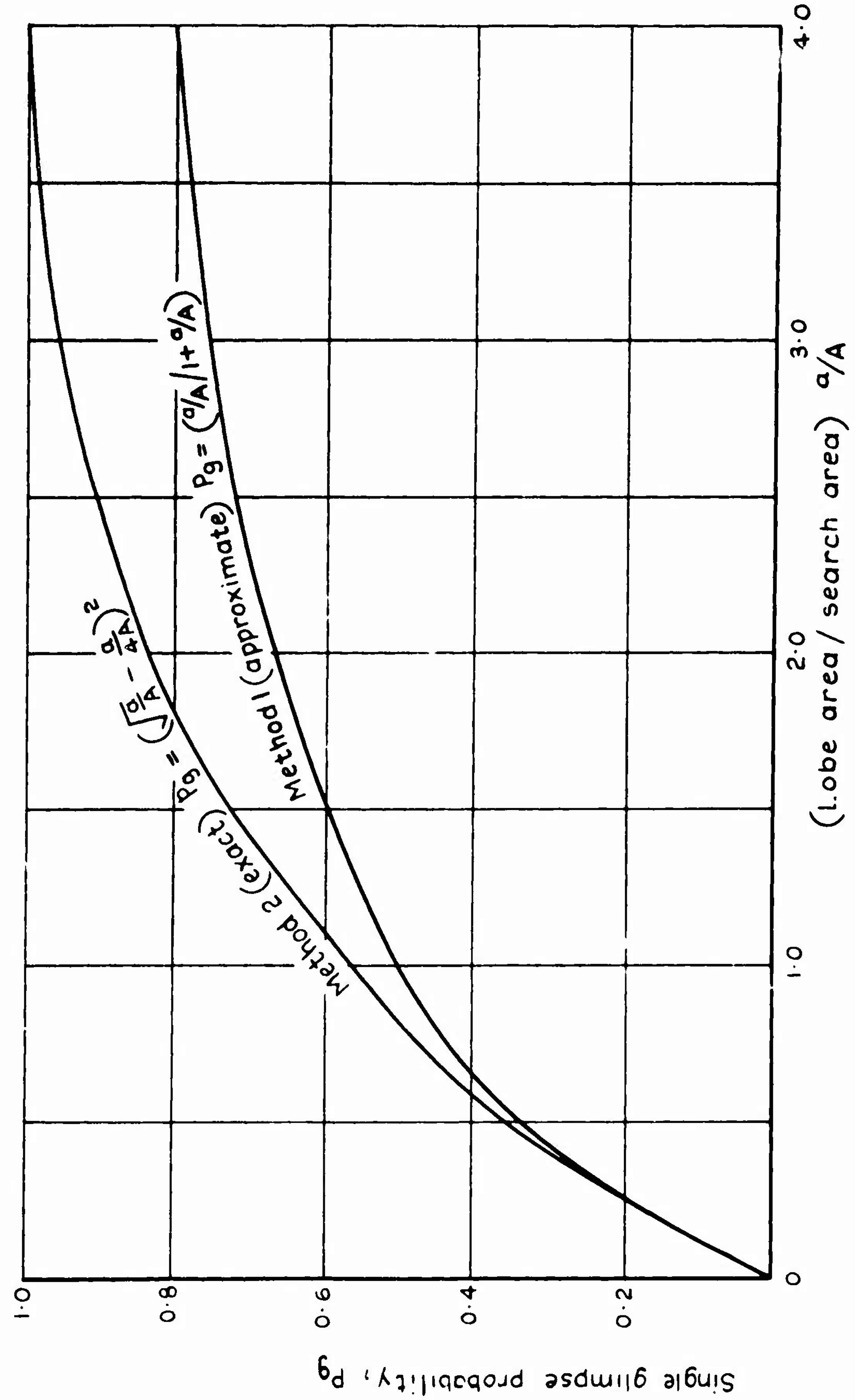


Fig. 1 Comparison between two methods of calculating glimpse probability

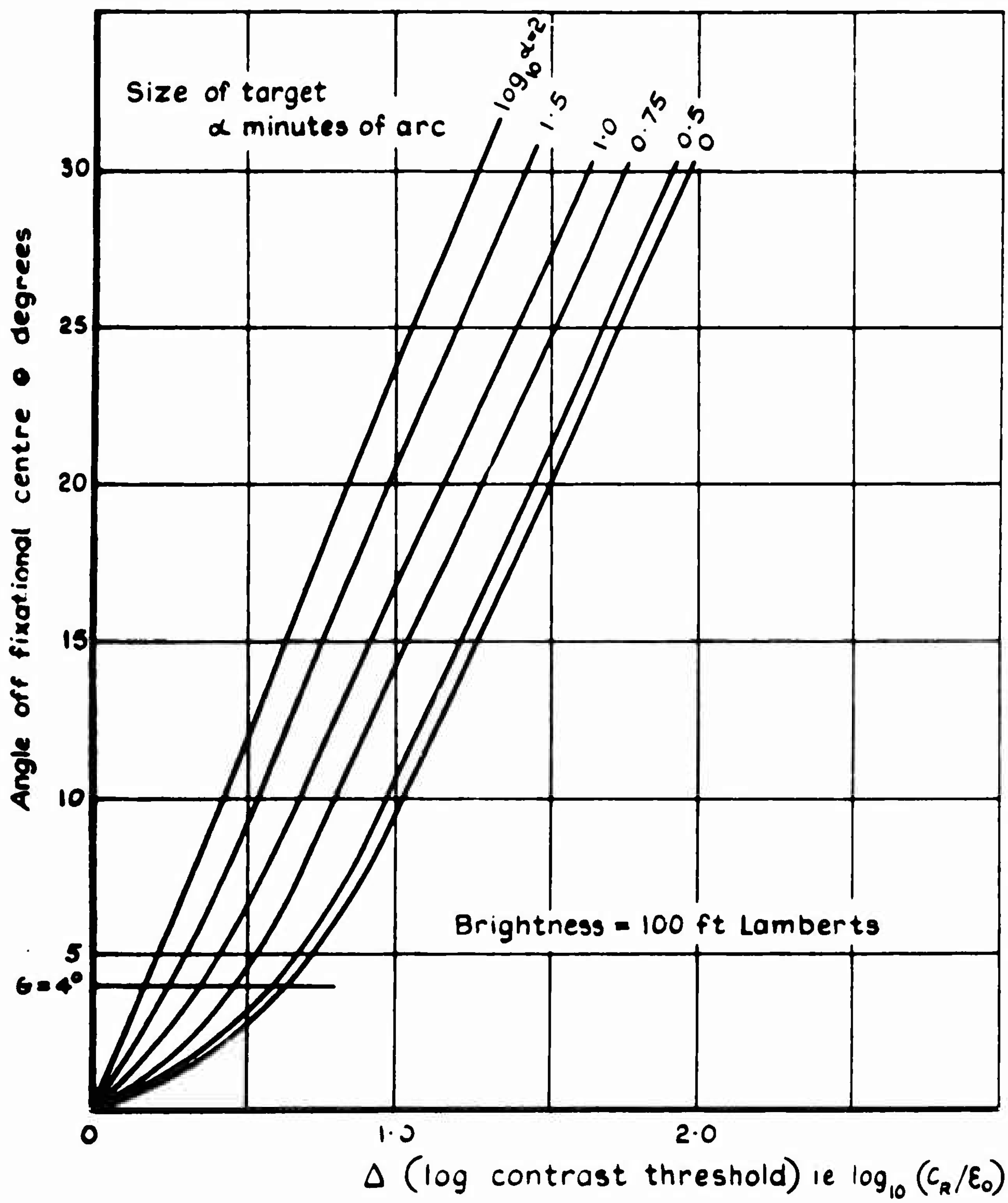


Fig. 2 J.H. Taylor's off-axis data for the eye

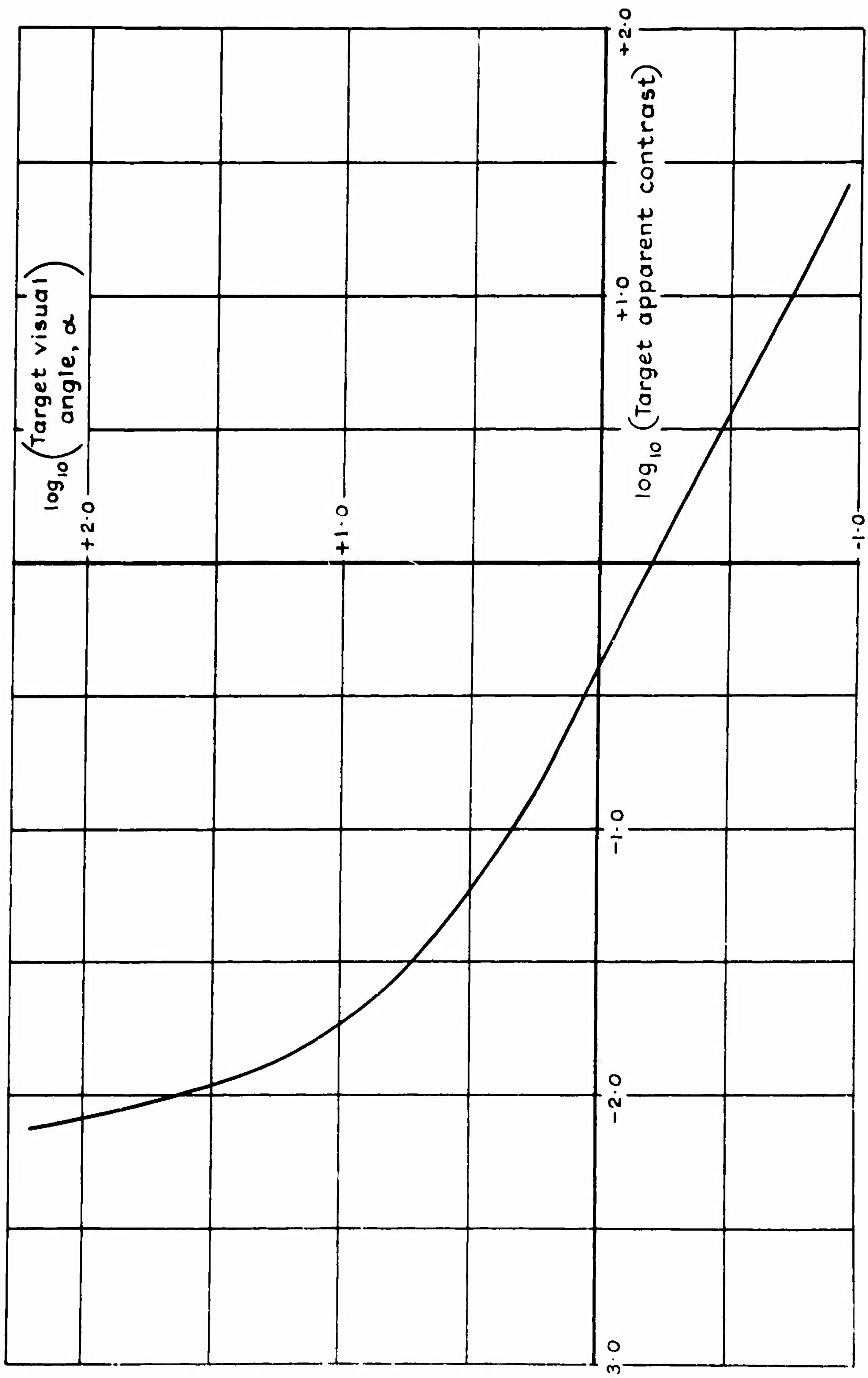


Fig 3 Blackwell 8 position search in 6 seconds threshold curve

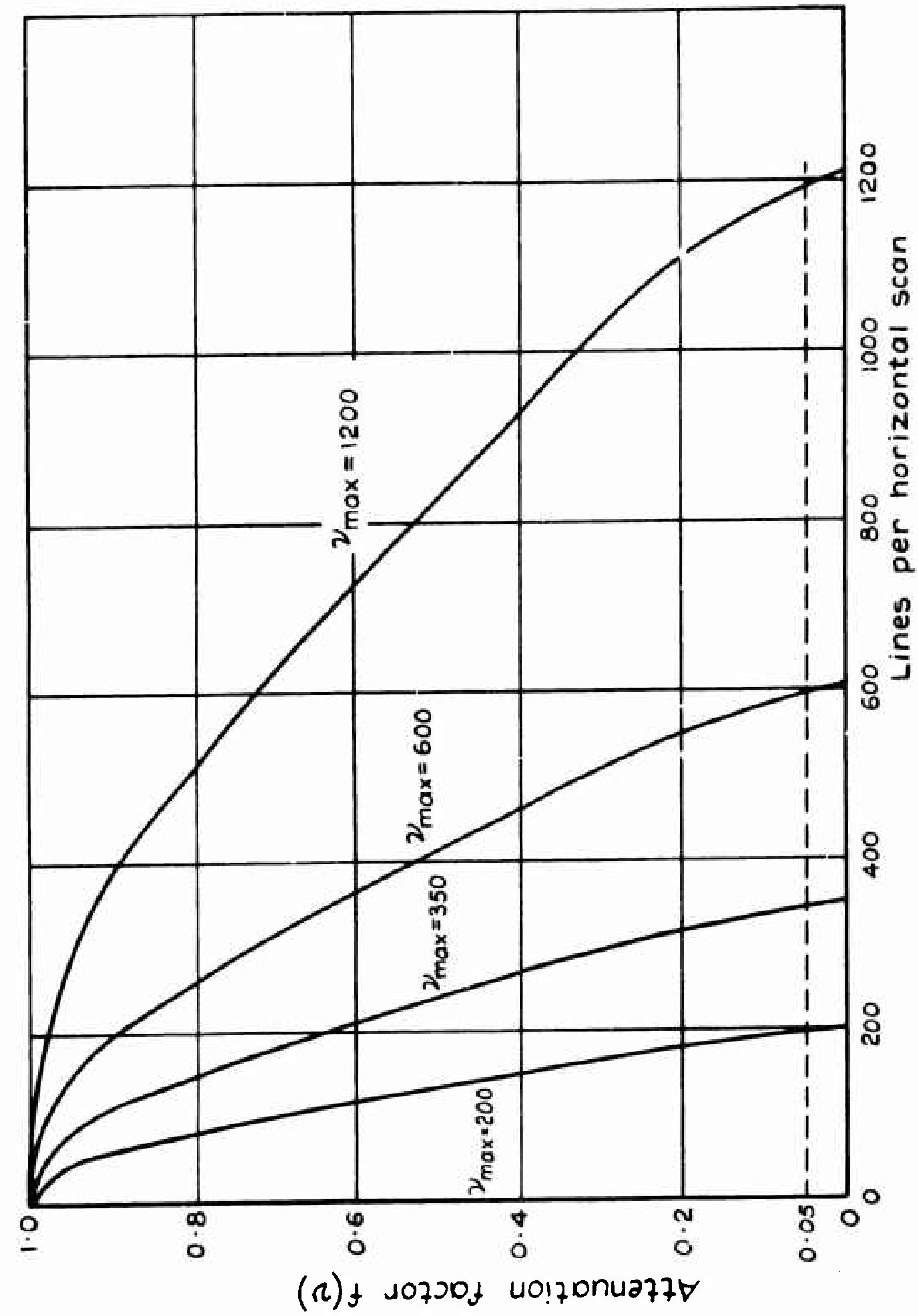


Fig. 4 Typical frequency response curves for television cameras

Fig. 5

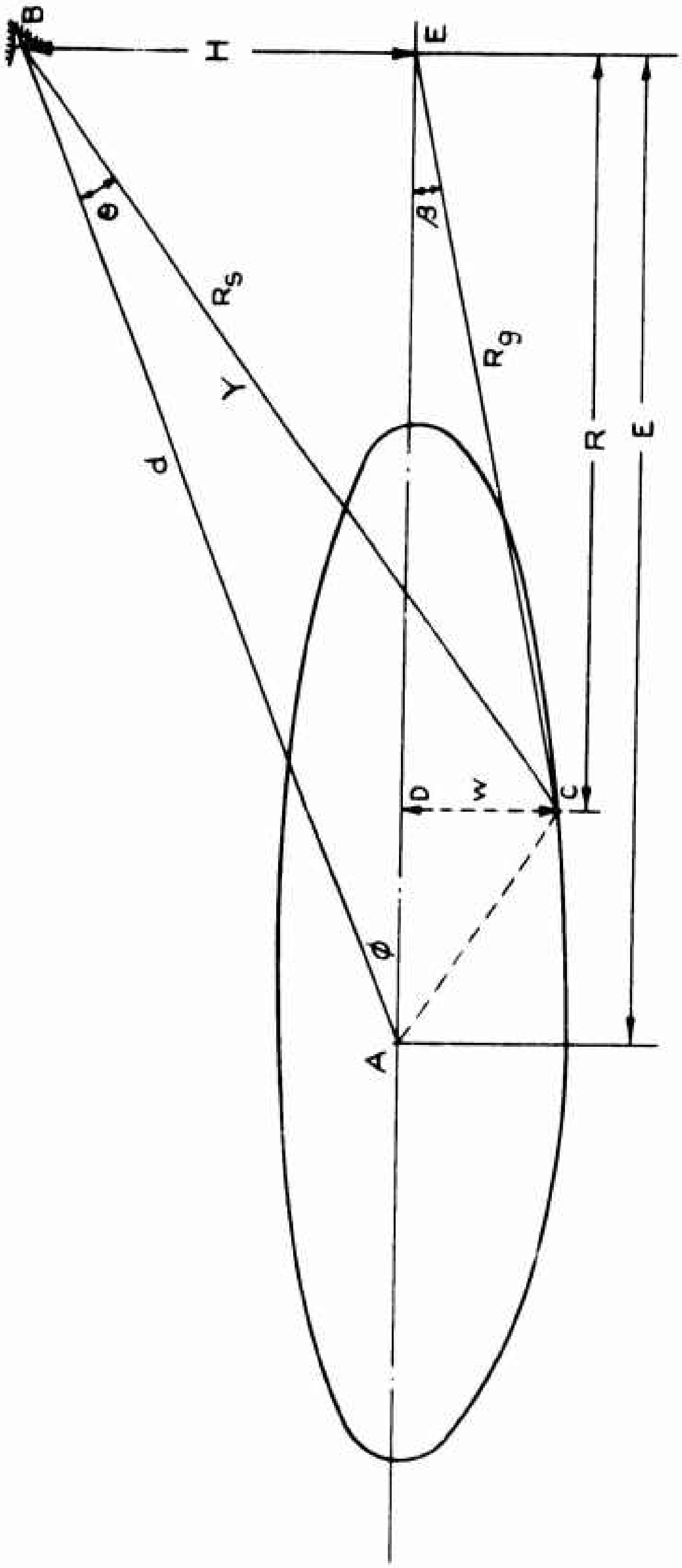


Fig.5 Geometry of a general point on the ground lobe section

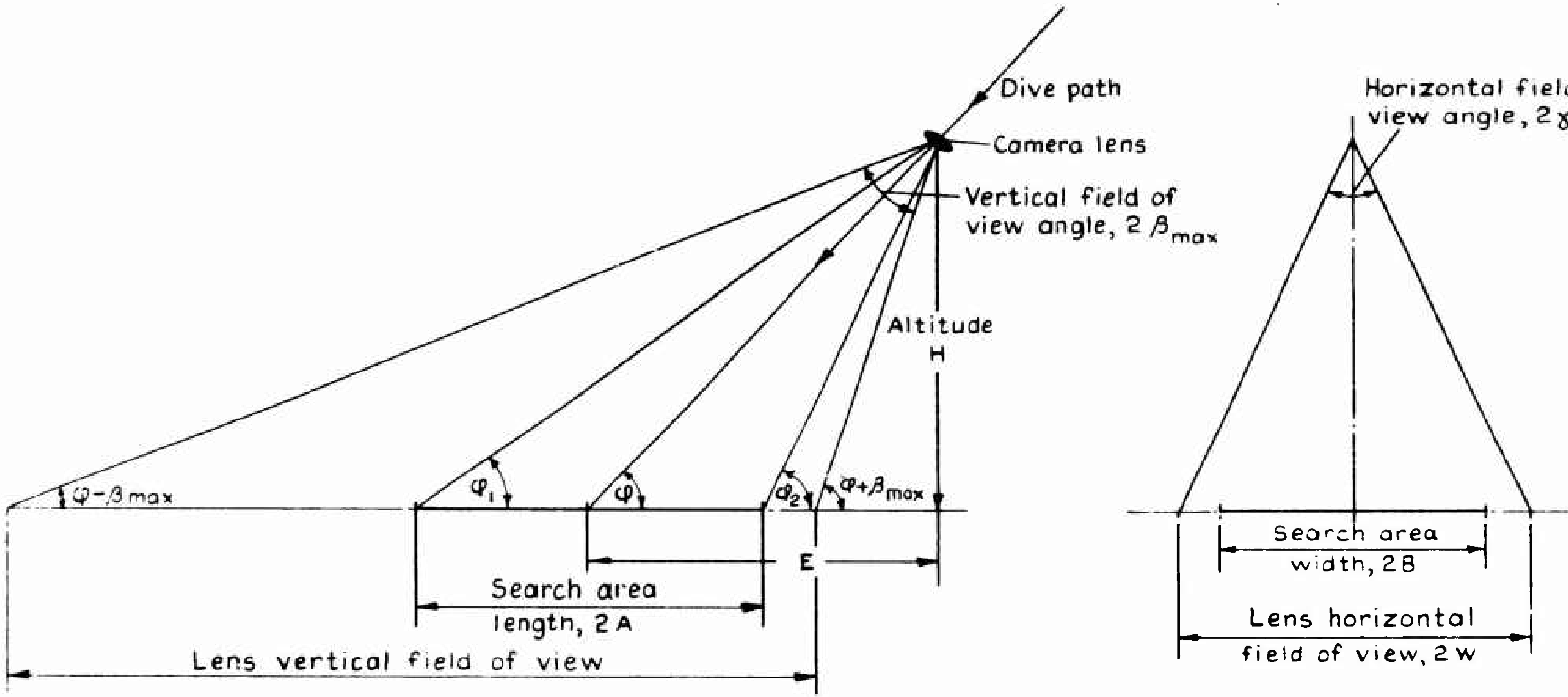


Fig. 6 Televisual viewing situation – fields of view and search area size

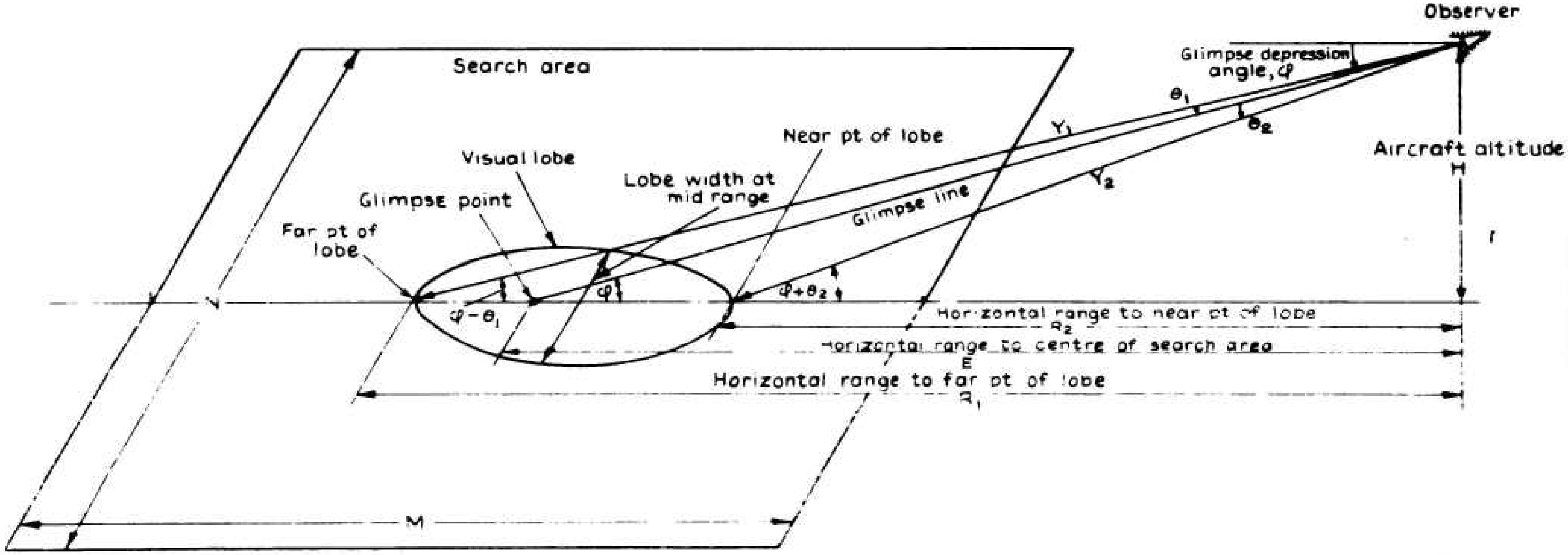


Fig. 7 General search situation with ground lobe section

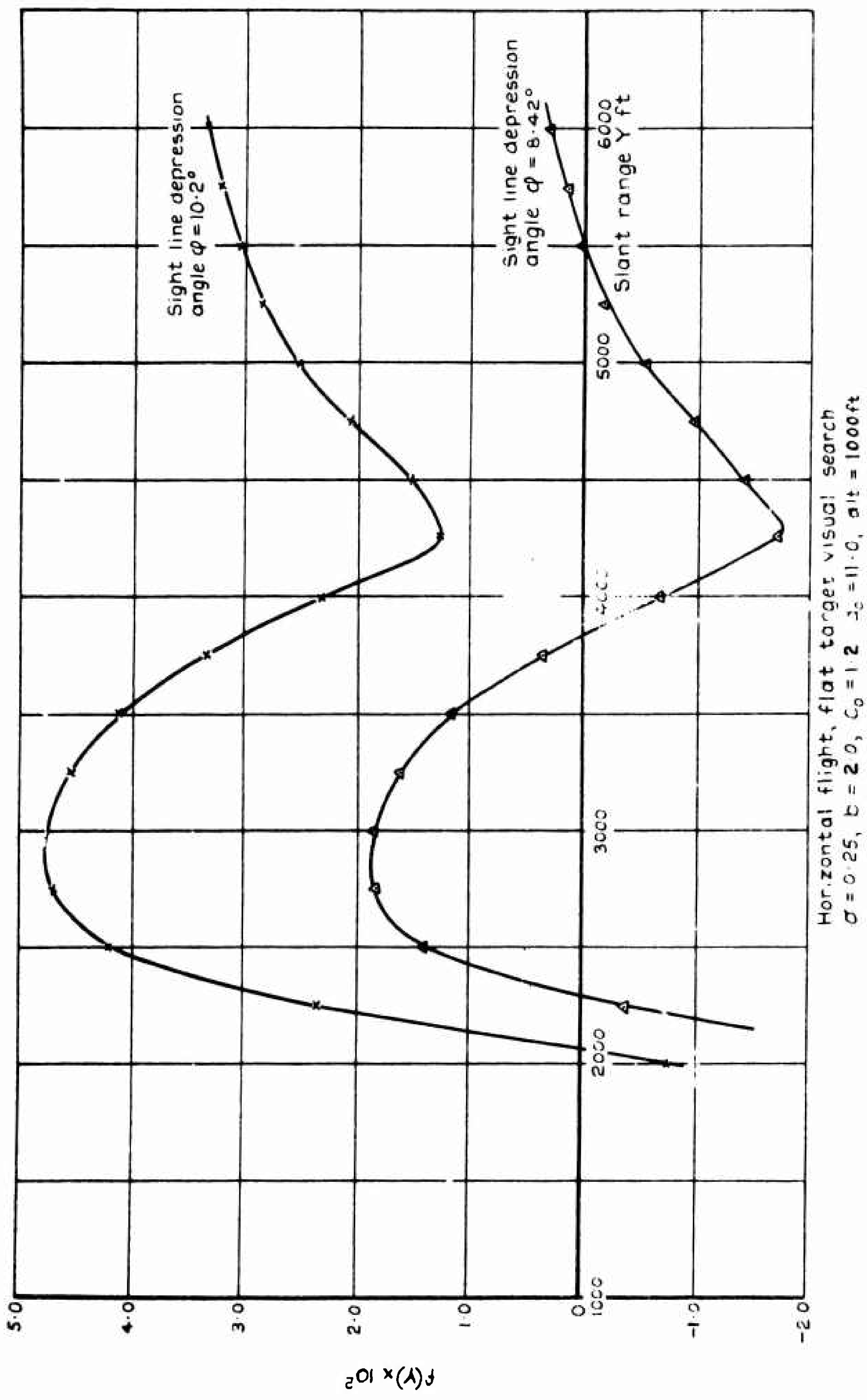
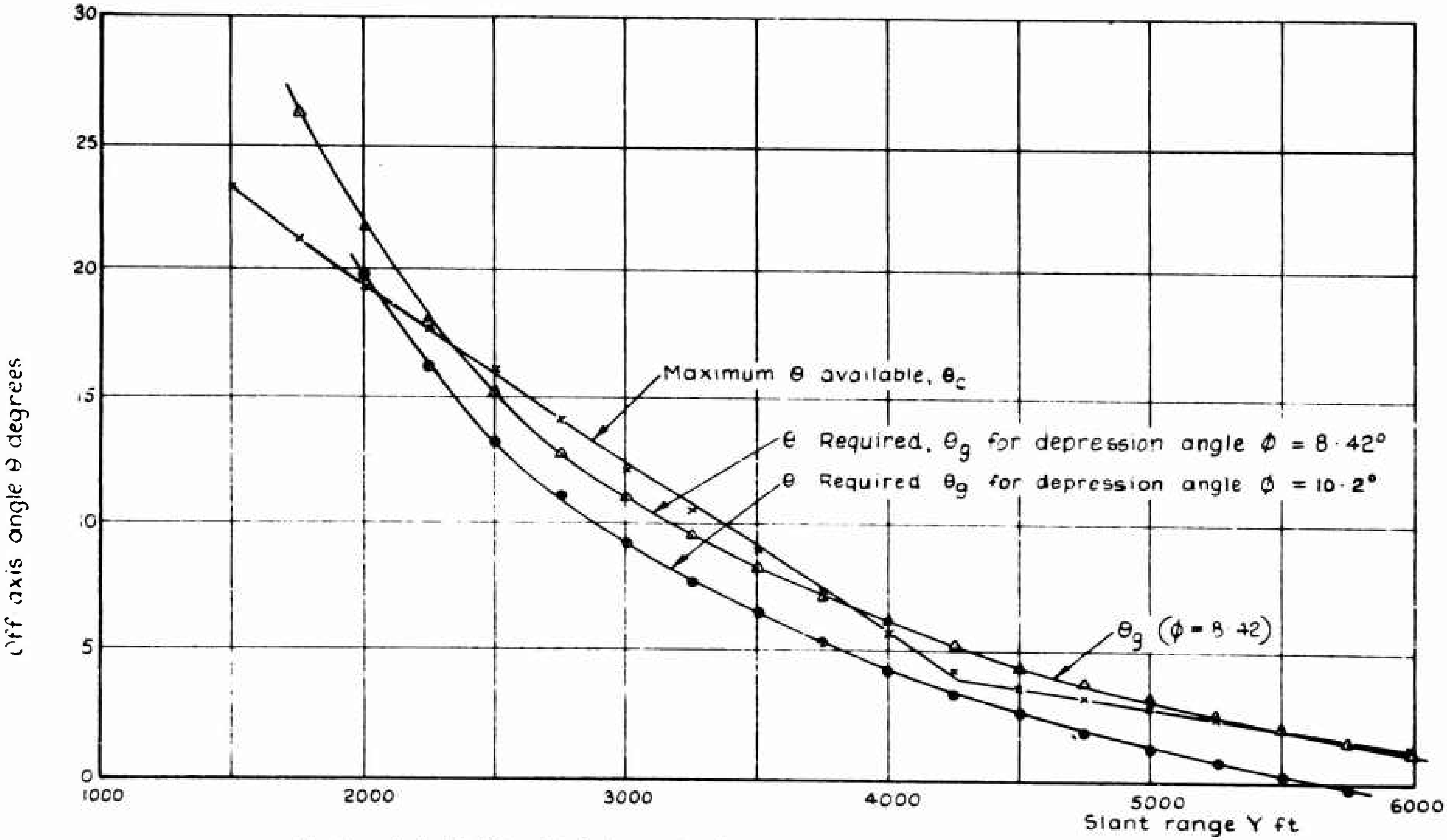


Fig. 8 An example of multiple solutions to the lobe near point slant range



Horizontal flight; Flat target visual search

$\sigma = 0{\cdot}25$, $b = 2{\cdot}0$, $C_o = 1{\cdot}2$ $d_o = 11{\cdot}0$ alt = 1000 ft

Fig.9 Off-axis angles available and required for solution of lobe near point

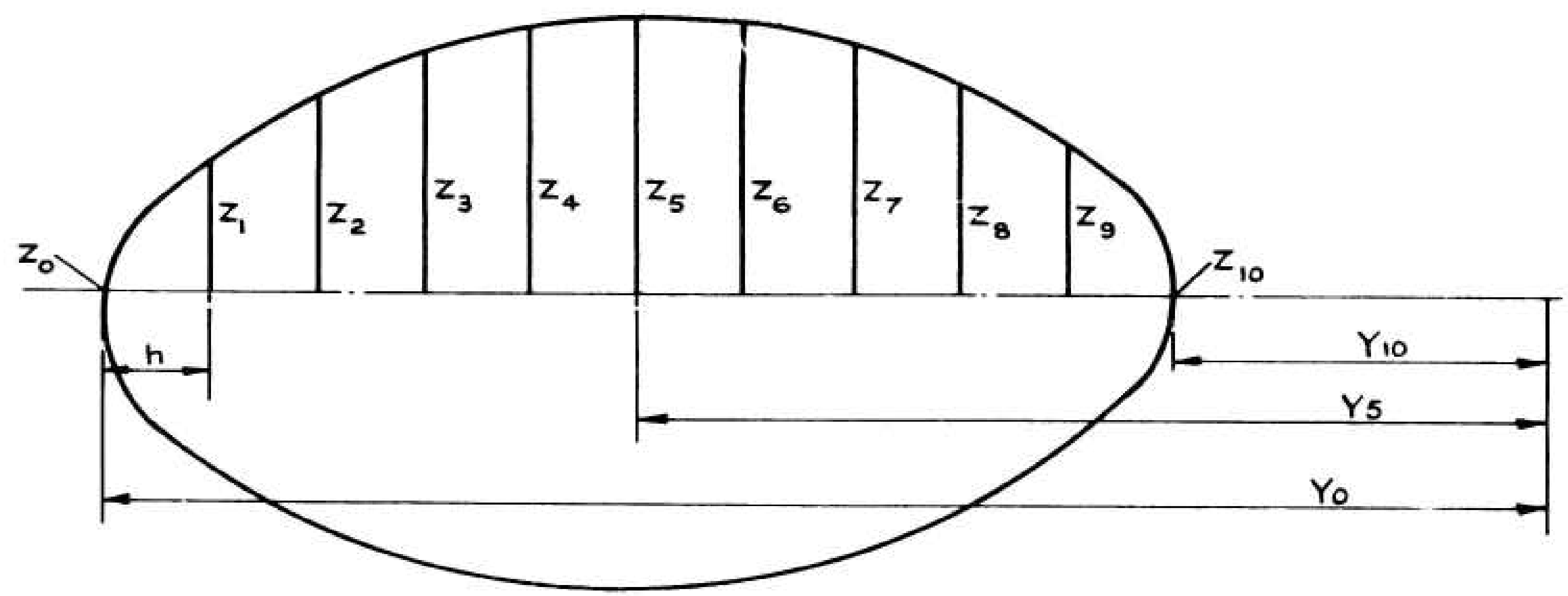
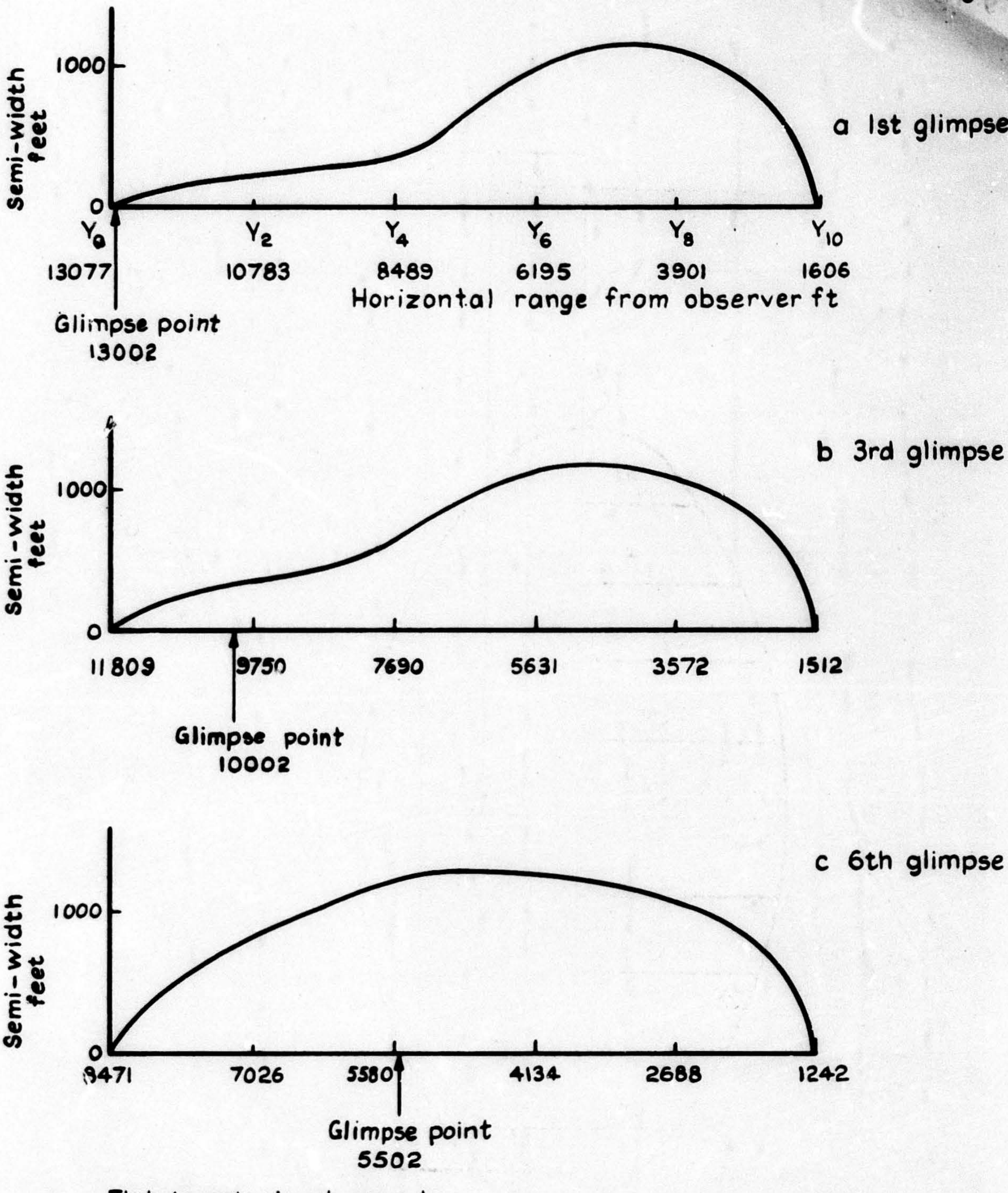


Fig. 10 Typical visual lobe divided into strips for calculating its area



Flat target visual search

$\sigma = 0{\cdot}25$, $b = 2{\cdot}0$, $\Delta t = 1{\cdot}5$ sec, $C_o = 1{\cdot}2$, $d_o = 34{\cdot}2$, altitude = 1000 ft
speed = 1000 ft/sec, search area = 10^6 ft^2

Fig. 11 Variation of lobe shape with distance between the observer and glimpse point in the search area

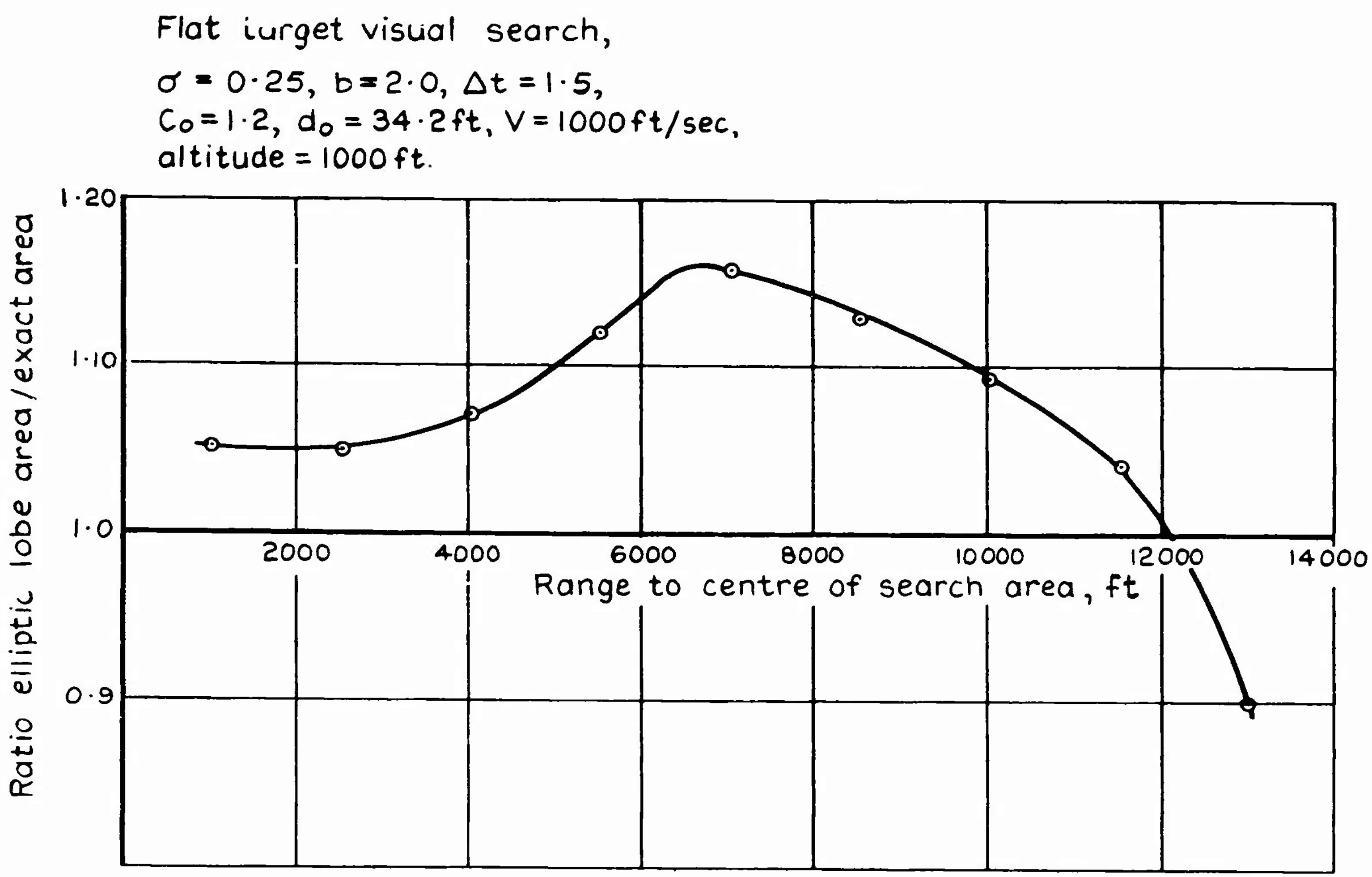


Fig. 12 Variation of ratio (elliptic lobe area/exact area) with distance between observer and search area

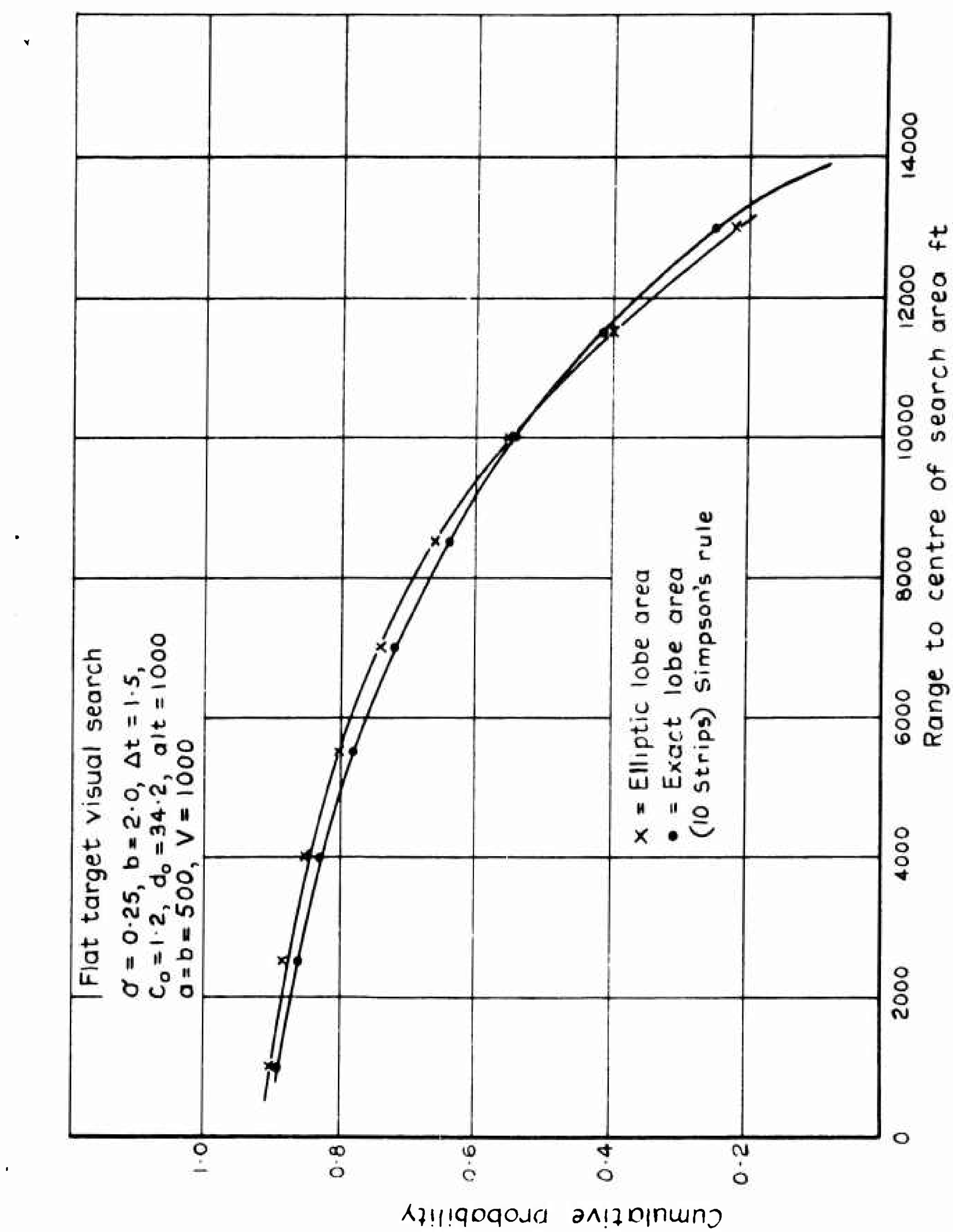


Fig. 13 Comparisons between cumulative glimpse probabilities based on exact and elliptic lobe areas

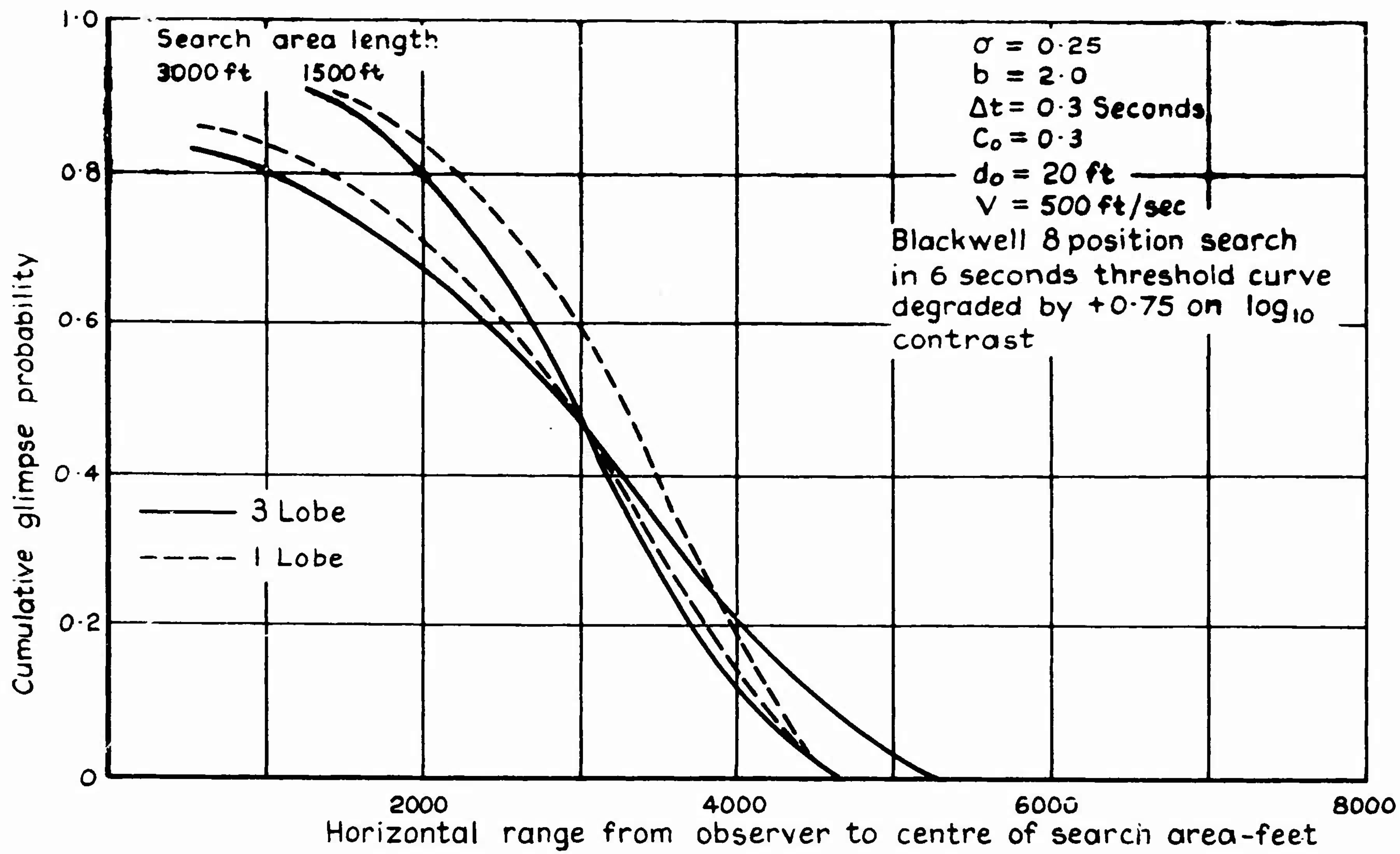


Fig. 14a Comparison between cumulative probability theory calculated using an average lobe size based on (a) Three sample lobes and (b) A single sample lobe. Height 500 ft

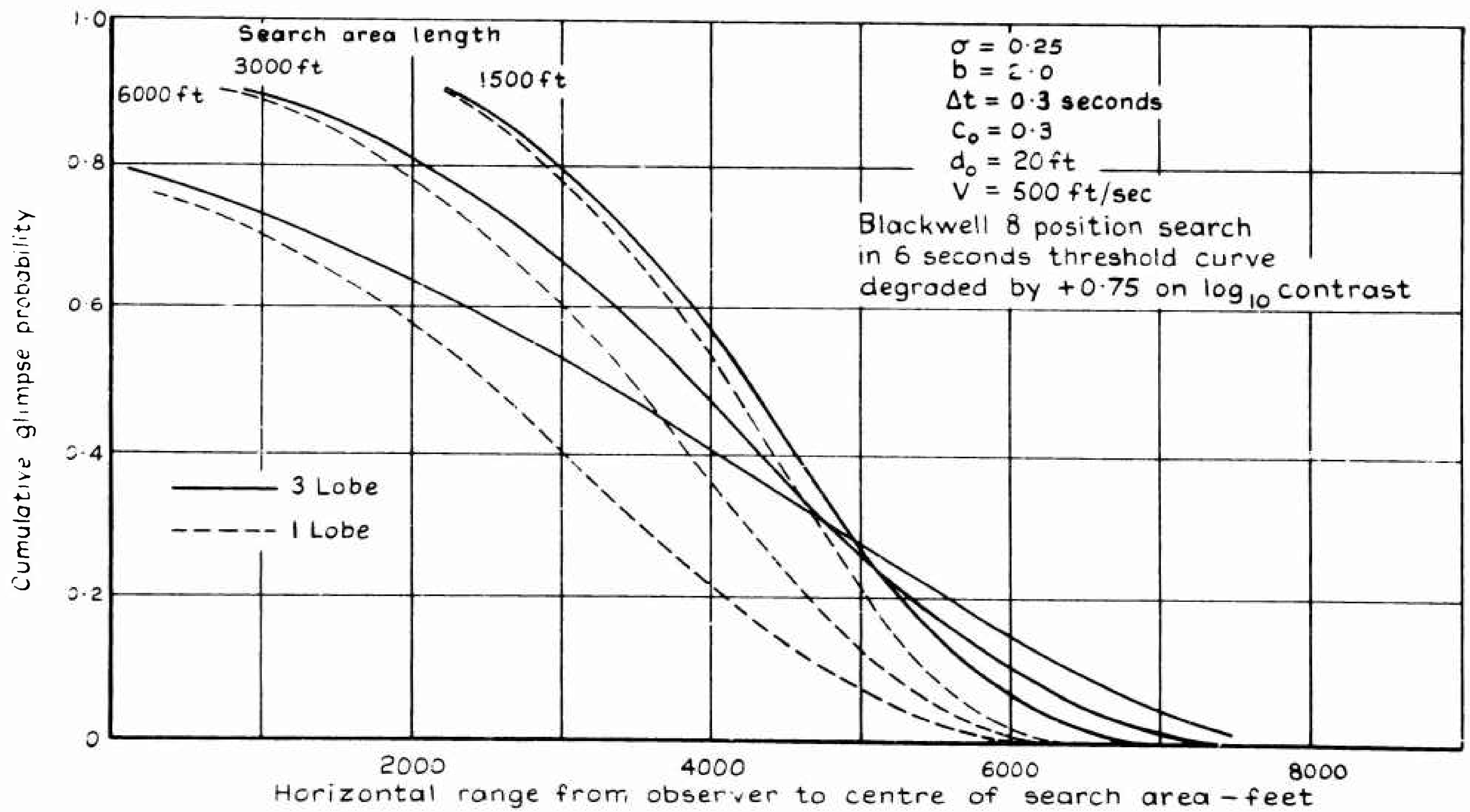


Fig. 14b Comparison between cumulative probability theory calculated using an average lobe size based on (a) Three sample lobes and (b) A single sample lobe. Height 2000 ft

DETACHABLE ABSTRACT CARDS

Smith, L.J. 621.396.969.3 :
Appendix by A.R. Runnalls 623.55.021

THE APPLICATION OF VISUAL LOBE SEARCH THEORY TO AIR TO GROUND TARGET DETECTION

Royal Aircraft Establishment Technical Report 68253 October 1968

Details are given of how a mathematical model, based on search theory and target size and contrast, can be programmed for a digital computer to enable the cumulative probability of detection of a ground target from the air to be determined as a function of range. This is applied to cover both visual and televisual viewing for flat targets and for targets of fixed presented area. An account of some of the difficulties involved is included, together with justification of some of the simplifications found possible.

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